Comparison of single EWMA-type control charts based on Economic-statistical design

Shin-Li Lu
Chen-Fang Tsai
Department of Industrial Management and Enterprise Information
Aletheia University, New Taipei City, Taiwan

Keywords
EWMA charts; Simulation; Economic-statistical design; Cost model; Loss function

Abstract

A single exponentially weighted moving average (EWMA) chart is effectively used for monitoring the process mean and variance simultaneously. In this paper, three single EWMA-type control charts including sum of square EWMA (SS-EWMA), maximum EWMA (MaxEWMA), and EWMA-semicircle (EWMA-SC) charts are compared under Lorenzen and Vance's cost model integrating Taguchi's loss function. The optimal decision variables, namely sample size n*, sampling interval time* h*, control limit width* L*, and smoothing constant* λ*, are obtained by minimizing the expected cost function. Via simulations, the EWMA-SC charts have the smallest expected cost among these charts when a process has large shifts. However, the MaxEWMA charts have the smallest expected cost among these charts when a process mean shifts.*

1. Introduction

The exponentially weighted moving average (EWMA) chart was first introduced by Roberts (1959) and has been widely used to improve the quality of a process when small process shifts are of interest. Various types of EWMA charts have been sequentially proposed to monitor process shifts. The variable sampling interval (VSI) EWMA chart (Saccucci et al., 1992) and the variable sample sizes and sampling intervals (VSSI) EWMA chart (Reynolds and Arnold, 2001) are efficient in improving detection ability than fixed-type EWMA charts. Auto correlated observations are always generated by continuous product manufacturing processes. Facing this scenario, it is not advisable to use traditional control chart for monitoring process shifts. Harris & Ross (1991) discussed the impact of autocorrelation on the performance of EWMA charts, and showed that the average run lengths of these charts were sensitive to the presence of autocorrelation. Most of control charts are established on the belief that the observations of a process are assumed to follow a normal distribution. When the observations are from a non-normal or unknown distribution, it is suitable to construct control charts based on a nonparametric approach. Bakir (2006) proposed the nonparametric EWMA control charts for monitoring an unspecified in-control target process center. The proposed charts are more efficient than traditional normal-based control charts.

Two EWMA charts are usually required to jointly monitor small process mean and variance shifts. However, recently, significant attempts have been made to design a single EWMA chart for monitoring the process mean and variance simultaneously. Researchers have presented designs of various single EWMA control charts, such as the MaxMin EWMA chart (Amin et al., 1999), the Max chart (Chen and Cheng, 1998), the sum of squares EWMA (SS-EWMA) chart (Xie, 1999), the maximum EWMA (MaxEWMA) chart (Chen et al., 2001), the EWMA-semicircle (EWMA-SC) chart (Chen et al., 2004). These charts transform the sample mean and sample variance into a single plotting statistic or two plotting statistics, one representing the mean and the other representing the variance, on the same chart.

The aforementioned control charts are designed from a statistical perspective. Statistically designed control charts are often measured using the desired in-control average run length (ARL_0) and out-of-control average run length (ARL_1). In addition to the statistical design of control charts, another design perspective involves economic design. The objective of an economic design control chart is to minimize the expected hourly loss cost. Duncan (1956) first proposed the economic model of a $\bar{X}$ control chart, wherein three parameters, namely sample size n, sampling interval h, and control limit width L must be determined by cost minimization. This approach was generalized by Lorenzen and Vance (1986), who considered whether production continues during the period of searching for and/or repairing the assignable cause. Since then, various economic designs have been proposed for control charts (Montgomery, 1980; Ho and Case, 1994a; Park et al., 2004; Chou et al., 2006). Recently, Serel and Moskowitz (2008) and Serel (2009) presented a cost-minimization model to design the EWMA control chart based on quality-related production costs using the loss function.

However, Woodall (1986) found that the optimal economic design control chart has poor statistical performance. One significant problem is that the optimal economic solution usually yields a considerably large risk of Type I error. The in-control ARL_0 of the control chart is usually too short to be practically acceptable. To improve the low statistical performance of the economic design control chart, some authors have expanded the pure economic model by incorporating additional statistical constraints, such as Saniga (1989), Montgomery et al. (1995), Chou et al. (2000) and Chen and Pao (2011), Yeong et al. (2013). Taguchi's quadratic loss function has been integrated into the economic-statistical design most recently by Huang and Lu (2015) and Lu et al. (2013), who respectively proposed the economic-statistical design of MaxEWMA and SSEWMA. They found that the economic-statistical design results in a large improvement in statistical performance and a small increase in cost.

The aim of this paper is to develop an economic-statistical design of the MaxDEWMA control chart by integrating the loss function into Lorenzen and Vance's cost model. A numerical simulation is conducted to minimize the cost function under ARL constraints. Moreover, a sensitivity analysis is conducted to assess the effects of the main input parameters on the objective function and decision variables. The rest of this paper is organized as follows. Section 2 reviews the literature on SS-EWMA, MaxEWMA and EWMA-SC charts. In Section 3, Lorenzen and Vance's cost model is briefly introduced. An illustrative example is presented in Section 4. Finally, Section 5 concludes.

2. Review of the SS-EWMA, MaxEWMA and EWMA-SC charts

Suppose X_{ij}, $i=1,2,\ldots$, and $j=1,2,\ldots,n_i$ be observations of size, n_i, in the i^{th} sample having a normal distribution with mean $\mu_0+\delta\sigma_0$ and standard deviation $\rho\sigma_0$, where μ_0 and σ_0 are defined as target values of the process. When $\delta=0$ and $\rho=1$ indicate that the process is in control, otherwise the process has changed or drifted.

Let $\bar{X}_i$ and S_i^2 denote the sample mean and sample variance of sample i, respectively. Then $\bar{X}_i$, $i=1,2,\ldots$ are independent normal random variables with mean $\mu_0+\delta\sigma_0$ and variance $\rho^2\sigma_0^2/n_i$; $(n_i-1)S_i^2/\rho^2\sigma_0^2$ and $i=1,2,\ldots$ are independent chi-square random variables with n_i-1 degrees of freedom; and $\bar{X}_i$ and S_i^2 are independent.

2.1 The SS-EWMA control chart

According to Xie (1999), define the following two statistics:

$$U_i = \frac{(\bar{X}_i - \mu_0)}{\sigma_0 / \sqrt{n_i}} \qquad (1)$$

and

$$V_i = \Phi^{-1}\left\{F\left[\frac{(n_i - 1)S_i^2}{\sigma_0^2}, n_i - 1\right]\right\}, \qquad (2)$$

where $\Phi(z) = P(Z \le z)$, $Z \sim N(0\ ,\ 1)$, Φ^{-1} is the inverse function of Φ, and $F(w,v) = P(W \le w|v)$, where $W \sim \chi^2(v)$. (These transformations and applications have been proposed by Quesenberry (1995))

Both U_i and V_i are independent standard normal random variables when the process in control, and that the distributions of U_i and V_i are both independent of the sample size n_i. Two EWMA statistics, each for the mean and variance, can be defined as

$$A_i = \lambda U_i + (1-\lambda)A_{i-1}\ ,\ 0 < \lambda \le 1\ ,\ i = 1, 2\ldots\ , \qquad (3)$$

and

$$B_i = \lambda V_i + (1-\lambda)B_{i-1}\ ,\ 0 < \lambda \le 1\ ,\ i = 1, 2\ldots\ , \qquad (4)$$

where A_0 and B_0 are the starting values, respectively. It is known that A_i and B_i are independent because U_i and V_i are independent, and when $\delta = 0$, $\rho = 1$, and $A_0 = B_0 = 0$, we have both $A_i \sim N(0\ ,\ \sigma_{A_i}^2)$ and $B_i \sim N(0\ ,\ \sigma_{B_i}^2)$, where $\sigma_{A_i}^2 = \sigma_{B_i}^2 = \frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]$. We then define the statistic of the SS-EWMA chart by combing the above two EWMA statistics defined as

$$SE_i = A_i^2 + B_i^2\ ,\ i = 1, 2, \ldots . \qquad (5)$$

Because the statistic SE_i is non-negative, the SS-EWMA chart has only an upper control limit (UCL), which is given by

$$UCL = E(SE_i) + L\sqrt{Var(SE_i)} \qquad (6)$$

where $E(SE_i)$ and $Var(SE_i)$ are the in-control mean and variance of the statistic SE_i, respectively. L is the control limit constant chosen to match the desired ARL_0. To achieve the desired ARL_0, the corresponding control limit constant L and the smoothing parameter λ is determined.

2.2 The MaxEWMA control chart

According to definition of Xie (1999), Chen et al. (2001) redefined the new MaxEWMA statistic defined as

$$M_i = \max\{|A_i|, |B_i|\} \qquad (7)$$

Because the statistic M_i is non-negative, the MaxEWMA chart has only an upper control limit (UCL), which is given by

$$UCL = E(M_i) + L\sqrt{Var(M_i)} \qquad (8)$$

where $E(M_i)$ and $Var(M_i)$ are the in-control mean and variance of the statistic M_i, respectively. L is the control limit constant chosen to match the desired ARL_0. To achieve the desired ARL_0, the corresponding control limit constant L and the smoothing parameter λ is determined.

2.3 The EWMA-SC control chart

According to Chen et al. (2004), the statistic of a SC chart is defined as:

$$H_i = (\bar{X}_i - \mu)^2 + \frac{n_i - 1}{n_i} S_i^2, \quad i = 1, 2, \ldots \quad (9)$$

Let $H_i^* = \frac{n_i}{\sigma^2} H_i$. The EWMA-SC statistic SC_i can be defined from H_i^* as follows:

$$SC_i = \lambda H_i^* + (1-\lambda) SC_{i-1}, \quad 0 < \lambda \le 1, \quad i = 1, 2, \ldots, \quad (10)$$

where λ is the smoothing constant while $SC_0 = n$ is the starting value of SC_i. Because $H_i^* \sim \chi^2(n)$ when $\delta = 0$, $\rho = 1$ and $n_1 = n_2 = \cdots = n_i = n$, and so we have the following results:

$$E(SC_i) = E(H_i^*) = n, \quad (11)$$

$$Var(SC_i) = \frac{\lambda\left[1-(1-\lambda)^{2i}\right]}{2-\lambda} \cdot Var(H_i^*) = \frac{2n\lambda\left[1-(1-\lambda)^{2i}\right]}{2-\lambda}. \quad (12)$$

In addition, Equation (10) can be rewritten as:

$$SC_i = Y_i + Z_i + n, \quad (13)$$

where $Y_i = \lambda\left[\frac{(\bar{X}_i - \mu)^2}{\sigma^2/n} - 1\right] + (1-\lambda)Y_{i-1}$ and $Z_i = \lambda\left[(n-1)(\frac{S_i^2}{\sigma^2} - 1)\right] + (1-\lambda)Z_{i-1}$ with $Y_0 = Z_0 = 0$. Additionally, it is known that Y_i and Z_i are also independent because $\bar{X}_i$ and S_i^2 are independent.

The EWMA-SC chart only needs an upper control limit (UCL) as the SC_i is non-negative. The UCL corresponding to Equation (10) is given by:

$$\text{UCL}^1 = E(SC_i) + L\sqrt{\text{Var}(\text{SC}_i)} = n + L\sqrt{\frac{2n\lambda\left[1-(1-\lambda)^{2i}\right]}{2-\lambda}}, \quad (14)$$

and the UCL corresponding to Equation (13) is given by:

$$\text{UCL}^2 = L\sqrt{\frac{2n\lambda\left[1-(1-\lambda)^{2i}\right]}{2-\lambda}}. \quad (15)$$

Here, L is the width of the control limits when the process is in the control state. The process is considered to be out of control whenever SC_i exceeds UCL^1 or (Y_i, Z_i) is outside the control region $\{(Y_i, Z_i): Y_i + Z_i \le \text{UCL}^2\}$, and some action should be taken. Indeed, the latter method of determining when a process is out of control is preferable, because the source of an assignable cause can be directly identified by plotting the location of the sample point on the chart.

3. Cost model

Lorenzen and Vance's cost model (1986) is employed for determining the optimal decision values of the economic-statistical design of control charts. Some underlying assumptions in that cost model are: (1) The production cycle length is defined as the time interval from the start of the in-control state to the elimination of an assignable cause for the out-of-control state. (2) The time between the occurrences of an assignable cause follows an exponential distribution with a mean of $1/\theta$ hours. (3) Once the process is out of control, intervention is required to adjust the process and return it to the initial state of statistical control. The expected cost per hour, denoted by $E(A)$, is derived by dividing the expected cost per cycle by the expected cycle length. Thus,

$$E(A)=\left\{\frac{a+bn}{h}\left(\frac{1}{\theta}+h\cdot ARL_1-\tau+nT_1+\gamma_2T_2+\gamma_3T_3\right)\right\}\div\left\{\frac{1}{\theta}+h\cdot ARL_1-\tau\right.$$
$$\left.+(1-\gamma_2)\frac{n_s\times T_0}{ARL_0}+nT_1+T_2+T_3\right\}+\left\{\frac{c_0}{\theta}+c_1(h\cdot ARL_1-\tau+nT_1+\gamma_2T_2+\gamma_3T_3)\right.$$
$$\left.+\frac{n_s\cdot c_2}{ARL_0}+c_3\right\}\div\left\{\frac{1}{\theta}+h\cdot ARL_1-\tau+(1-\gamma_2)\frac{n_s\times T_0}{ARL_0}+nT_1+T_2+T_3\right\}$$

where,

n = sample size,

h = time interval between samples,

λ = smoothing constant of the control chart,

L = control limit constant of the control chart,

τ = expected time between an assignable cause and the prior sample, denoted by $[1-(1+\theta h)e^{-\theta \cdot h}]/\theta(1-e^{-\theta \cdot h})$,

n_s = expected number of samples taken while in control, denoted $e^{-\theta h}/(1-e^{-\theta h})$,

μ_0 = target process mean,

σ_0 = target process standard deviation,

T_0 = expected time to search a false alarm,

T_1 = expected time to sample, inspect and plot each sample unit,

T_2 = expected time to search the assignable cause,

T_3 = expected time to repair the assignable cause,

γ_2 =1 if production continuous during searches,
=0 if production ceases during searches,

γ_3 =1 if production continuous during repair,
=0 if production ceases during repair,

a = fixed cost of sampling,

b = unit variable cost of sampling,

c_0 = the expected quality loss per unit of product when the process is in control,

c_1 = the expected quality loss per unit of product when the process is out of control,

c_2 = cost of investigating a false alarm,

c_3 = cost of searching and repairing an assignable cause.

Traditionally, a product is classified as either conforming or nonconforming according to the specifications of the relevant quality characteristic. Quality costs are consequently incurred when products fall outside the specification limits. However, in practice, a product's performance will gradually deteriorate as the design parameter deviates from the target value. A loss function was first introduced by Taguchi (1985) to describe the quality characteristic differs from the nominal. The loss refers to the quality cost that is incurred when the quality characteristic is off target even though it may conform to the specification limits. Taguchi's loss function has been broadly employed in industrial applications, especially in the economic or economic-statistical designs of control charts (Serel and Moskowitz, 2008; Serel, 2009; Yeong et al., 2013; Lu et al., 2013). The economic-statistical designs of control charts based on Taguchi's loss function are investigated in this paper.

Since the symmetric loss function is more common in applications, in this paper, we consider symmetric loss functions and the constant loss coefficient K such that the quadratic function is presented as

$$L_Q(x) = K(x-T)^2, \tag{16}$$

where the quality characteristic x has a probability density function $f(x)$, the target value T is a parameter describing the risk aversion of the decision makers.

When the process is in control J_{0Q} is the expected loss per unit of product and we denote it as follows:

$$J_{0Q} = \int_{-\infty}^{\infty} L_Q(x) f(x)dx = K[\sigma_0^2 + (\mu_0 - T)^2] \tag{17}$$

When the process is out of control, the process mean shifts to $\mu_1 = \mu_0 + \delta\sigma_0$ and/or the process variance shifts to $\sigma_1 = \rho\sigma_0$. The expected loss per unit of product is named J_{1Q} and represented as follows:

$$J_{1Q} = K[\sigma_1^2 + (\mu_0 - T)^2 + \delta^2\sigma_0^2 - 2\delta\sigma_0(\mu_0 - T)] \tag{18}$$

Assume the production rate to be P units per hour. The quality costs c_0 and c_1 in Lorenzen and Vance's cost model are replaced with the two expected product losses $J_{0Q}P$ and $J_{1Q}P$, respectively.

Consider the cost model integrating the loss function, wherein quality costs c_0 and c_1 are replaced by $J_{0Q}P$ and $J_{1Q}P$, respectively. Consequently, the expected cost (or loss) per hour, denoted by $E(\tilde{A})$, may be expressed as

$$\min E(\tilde{A}) = \left\{ \frac{a+bn}{h}\left(\frac{1}{\theta} + h\cdot ARL_1 - \tau + nT_1 + \gamma_2 T_2 + \gamma_3 T_3\right) \right\} \div \left\{ \frac{1}{\theta} + h\cdot ARL_1 - \tau \right.$$
$$\left. + (1-\gamma_2)\frac{n_s \times T_0}{ARL_0} + nT_1 + T_2 + T_3 \right\} + \left\{ \frac{J_{0Q}P}{\theta} + J_{1Q}P(h\cdot ARL_1 - \tau + nT_1 + \gamma_2 T_2 + \gamma_3 T_3) \right.$$
$$\left. + \frac{n_s \cdot c_2}{ARL_0} + c_3 \right\} \div \left\{ \frac{1}{\theta} + h\cdot ARL_1 - \tau + (1-\gamma_2)\frac{n_s \times T_0}{ARL_0} + nT_1 + T_2 + T_3 \right\}$$

subject to

$$ARL_0 \geq ARL_0^*$$

$$n \in I^+, h, L \in R^+, 0 < \lambda \leq 1$$

Not only is the objective function $E(\tilde{A})$ a function of ARL_0 and ARL_1, but ARL_0 and ARL_1 are functions of the charting parameters of the control charts. Hence, the optimal decision variables $(n^*, h^*, L^*, \lambda^*)$ of the economic-statistical design of the control charts based on loss functions are determined by minimizing the objective function $E(\tilde{A})$. Table 1 presents the algorithmic description used to solve the objective functions of the economic-statistical designs of the control charts based on loss functions.

4. An example

The minimal expected cost and corresponding optimal decision variables are compared among the SS-EWMA, MaxEWMA, and EWMA-SC charts based on quadratic loss function. For this illustration, the following parameter values are employed to demonstrate the optimal economic-statistical design of the three charts: $a=5$, $b=1$, $c_2=300$, $c_3=150$, $\theta=0.01$, $K=1, T_0=2$, $T_1=0.5$, $T_2=2$, $T_3=0$ $\delta\in\{0,0.5,1,2\}$, $\rho\in\{1,1.5,2,3\}$, $\gamma_2=1$, $\gamma_3=0$, $r=1$, and $P=300$. For the desired in-control process, ARL_0 is set to 370. The optimal decision variables (n^*,h^*,L^*,λ^*) and optimal values of the economic-statistical design for various SS-EWMA, MaxEWMA, and EWMA-SC charts based on loss function are summarized in Tables 1-3.

Table 1. The optimal economic-statistical design of SS-EWMA charts under quadratic loss function

		$\rho=1.00$	$\rho=1.50$	$\rho=2.00$	$\rho=3.00$
$\delta=0.00$	n^*	-	5	2	2
	λ^*	-	0.35	0.84	0.91
	h^*	-	0.76	0.47	0.48
	L^*	-	4.765	4.908	4.910
	ARL_1	-	6.475	4.760	2.035
	$E(\tilde{A})^{min}$	-	346.60	360.84	404.11
$\delta=0.50$	n^*	7	4	2	2
	λ^*	0.19	0.42	0.80	0.90
	h^*	1.88	0.72	0.48	0.48
	L^*	4.476	4.814	4.909	4.991
	ARL_1	7.381	5.332	4.288	1.994
	$E(\tilde{A})^{min}$	319.69	346.25	362.44	406.36
$\delta=1.00$	n^*	4	3	2	2
	λ^*	0.34	0.51	0.72	0.91
	h^*	1.06	0.67	0.50	0.47
	L^*	4.750	4.854	4.902	4.910
	ARL_1	3.888	3.700	3.333	1.886
	$E(\tilde{A})^{min}$	331.76	350.56	367.74	413.12
$\delta=2.00$	n^*	2	2	2	2
	λ^*	0.58	0.70	0.77	0.91
	h^*	0.63	0.57	0.53	0.46
	L^*	4.880	4.901	4.908	4.910
	ARL_1	2.269	2.141	1.940	1.589
	$E(\tilde{A})^{min}$	361.21	374.73	392.29	440.14

Table 2. The optimal economic-statistical design of MaxEWMA charts under quadratic loss function

		$\rho=1.00$	$\rho=1.50$	$\rho=2.00$	$\rho=3.00$
$\delta=0.00$	n^*	-	5	3	2
	λ^*	-	0.30	0.73	0.88

	h^*	-	0.74	0.58	0.46
	L^*	-	3.347	3.436	3.444
	ARL_1	-	6.834	3.709	2.140
	$E(\tilde{A})^{min}$	-	347.44	362.34	405.21
$\delta = 0.50$	n^*	7	4	2	2
	λ^*	0.19	0.39	0.71	0.90
	h^*	1.92	0.69	0.46	0.46
	L^*	3.239	3.389	3.438	3.445
	ARL_1	7.098	5.878	4.617	2.092
	$E(\tilde{A})^{min}$	**319.40**	347.74	363.81	407.21
$\delta = 1.00$	n^*	4	3	2	2
	λ^*	0.35	0.48	0.73	0.90
	h^*	1.08	0.65	0.48	0.46
	L^*	3.374	3.412	3.440	3.445
	ARL_1	3.726	3.943	3.540	1.971
	$E(\tilde{A})^{min}$	**331.31**	351.53	368.86	414.11
$\delta = 2.00$	n^*	2	2	2	2
	λ^*	0.76	0.77	0.82	0.90
	h^*	0.65	0.57	0.51	0.45
	L^*	3.441	3.442	3.443	3.445
	ARL_1	2.166	2.158	2.012	1.645
	$E(\tilde{A})^{min}$	**360.48**	374.87	393.02	440.99

Table 3. The optimal economic-statistical design of EWMA-SC charts under quadratic loss function

		$\rho = 1.00$	$\rho = 1.50$	$\rho = 2.00$	$\rho = 3.00$
$\delta = 0.00$	n^*	-	3	2	2
	λ^*	-	0.15	0.23	0.41
	h^*	-	0.67	0.55	0.51
	L^*	-	3.110	3.617	4.194
	ARL_1	-	6.525	3.669	1.864
	$E(\tilde{A})^{min}$	-	**340.74**	**356.32**	**402.24**
$\delta = 0.50$	n^*	7	3	2	2
	λ^*	0.05	0.17	0.25	0.44
	h^*	1.12	0.68	0.55	0.50
	L^*	2.288	3.204	3.696	4.270
	ARL_1	24.627	5.351	3.424	1.835
	$E(\tilde{A})^{min}$	330.80	**342.93**	**358.52**	**404.56**
$\delta = 1.00$	n^*	5	2	2	2
	λ^*	0.16	0.19	0.30	0.47
	h^*	0.94	0.56	0.55	0.50
	L^*	3.024	3.444	3.874	4.339
	ARL_1	5.732	4.659	2.870	1.753
	$E(\tilde{A})^{min}$	337.77	**349.45**	**365.08**	**411.50**
$\delta = 2.00$	n^*	2	2	2	2

λ^*	0.30	0.38	0.43	0.54
h^*	0.62	0.58	0.54	0.48
L^*	3.874	4.115	4.245	4.481
ARL_1	2.361	2.080	1.849	1.521
$E(\tilde{A})^{\min}$	361.84	**374.22**	**391.34**	**439.06**

5. Conclusion

A single EWMA control chart has good statistical performance in detecting both the mean and the variance shifts simultaneously. It only measures control chart from statistical performance viewpoint. Control charts based on pure economically are unable to satisfy requests in practice when they just pay attention to minimization quality costs but neglect the high false alarm rate. Therefore, this work investigates an economic-statistical design of SS-EWMA, MaxEWMA and EWMA-SC control charts for monitoring process mean and/or variance by incorporating the Taguchi's quadratic loss function into Lorenzen and Vance's cost model. Numerical simulations are conducted to evaluate effects of main input factors on the optimal economic-statistical design of these three control charts.

Numerical simulations reveal that the optimal sample size n^*, sampling interval h^* and out-of-control ARL_1 decrease as the magnitude of mean and/or variance shifts increases, obviously in small process shifts. However, the optimal control limit L^* and smoothing constant λ^* increase as optimal value of $E(\tilde{A})^{\min}$ increases. Moreover, it is reasonable that the optimal value of $E(\tilde{A})^{\min}$ increases as the mean shift δ and/or variance shift ρ become large. Moreover, the MaxEWMA charts have the minimal cost when a process just has shifts in the mean. Once a process has shifts caused from the process variability, the EWMA-SC charts need minimal cost among these three charts.

References

Amin RW, Wolff H, Besenfelder W, Baxley RJR. (1999) EWMA control charts for the smallest and largest observations, Journal of Quality Technology. 31: 189-206.

Bakir ST. (2006) Distribution free quality control charts based on signed rank like statistics, Communications in Statistics-Theory and Method. 35:743-757.

Chen G, Cheng SW. (1998) Max chart: combining X-bar chart and S chart, Statistica Sinica. 8: 263-271.

Chen G, Cheng SW, Xie H. (2001) Monitoring process mean and variability with one EWMA chart, Journal of Quality Technology. 33: 223-233.

Chen G, Cheng SW, Xie H. (2004) A new EWMA control chart for monitoring both location and dispersion, Quality Technology and Quantitative Management. 1: 217-231.

Chen H, Pao Y. (2011) The joint economic-statistical design of $\bar{X}$ and R charts for nonnormal data, Quality and Reliability Engineering International. 27: 269-280.

Chou CY, Chen CH, Liu HR. (2000) Economic-statistical design of $\bar{X}$ charts for non-normal data by considering quality loss, Journal of Applied Statistic. 27: 939-951.

Chou CY, Chen CH, Liu HR. (2006) Economic Design of EWMA Charts with Variable Sampling Intervals, Quality and Quantitative. 40: 879-896.

Duncan AJ. (1956) The economic design of $\bar{X}$ charts used to maintain current control of a process, Journal of the American Statistical Association. 51: 228-242.

Harris TJ, Ross WH. (1991) Statistical process control procedures for correlated observations, Canadian Journal of Chemical Engineering. 69: 48-57.

Huang CJ, Lu SL. (2015) Considering Taguchi loss function on statistically constrained economic sum of squares exponentially weighted moving average charts, Journal of Statistical Computation and Simulation. 85: 572-586.

Ho C, Case K. (1994a) Economic design of control charts: A literature review for 1981-1991, Journal of Quality Technology. 26: 39-53.

Lorenzen TJ, Vance LC. (1986) The economic design of control charts: a unified approach, Technometrics. 28: 3-10.

Lu SL, Huang CJ, Chiu WC. (2013) Economic-statistical design of maximum exponentially weighted moving average control charts, Quality and Reliability Engineering International. 29:1005-1014.

Montgomery DC. (1980) The economic design of control charts: a review and literature survey, Journal of Quality Technology. 12: 75-87.

Montgomery DC, Torng CC, Cochran JK, Lawrence FP. (1995) Statistically constrained economic design of the EWMA control chart, Journal of Quality Technology. 27: 250-256.

Park C, Lee J, Kim Y. (2004) Economic design of a variable sampling rate EWMA chart, IIE Transactions. 36: 387-399.

Quesenberry CP. (1995) On properties of Q charts variables, Journal of Quality Technology. 27: 184-203.

Roberts SW. (1959) Control chart test based on geometric moving averages, Technometrics. 1: 239-250.

Reynolds MR, Arnold JC. (2001) EWMA control charts with variable sample sizes and variable sampling intervals, IIE Transactions. 33: 511-530.

Saccucci MS, Amin RW, Lucas JM. (1992), Exponentially weighted moving average control schemes with variable sampling intervals, Communications in Statistics-simulation and Computation. 21: 627-57.

Saniga EM. (1989) Economic statistical control chart designs with an application to $\bar{X}$ and R charts, Technometrics. 31: 313-320.

Serel DA. (2009) Economic design of EWMA control charts based on loss functions. Mathematical and Computer Modelling 49: 745-759.

Serel DA, Moskowitz H. (2008) Joint economic design of EWMA control charts for mean and variance, European Journal of Operational Research. 184: 157-168.

Taguchi G, Wu Y. (1985) Introduction to off-line quality control. Central Japan Quality Control Association, Tokyo.

Woodall WH. (1986) Weaknesses of the economic design of control charts, Technometrics. 28: 408-409.

Yeong WC, Khoo BC, Lee MH, Rahim MA. (2013) Economic and economic statistical designs of the synthetic $\bar{X}$ chart using loss functions, European Journal of Operational Research. 228: 571-581.

Xie, H. “Contributions to qualimetry”. Ph.D. Thesis, 1999, University of Manitoba, Winnipeg, Canada.