A Rolling Grey EWMA chart for improving process quality

Chen-Fang Tsai
Shin-Li Lu
Department of Industrial Management and Enterprise Information
Aletheia University, New Taipei, Taiwan

Keywords
Grey control chart; Average run length; Grey theory.

Abstract
A traditional assumption in statistical process control (SPC) is that the observations of the process are independent. However, the independence assumption is frequently not reasonable in many manufacturing processes, such as High-tech and the chemical industries. Some authors use standard control charts but adjust the control limits to account for autocorrelation when there is significant autocorrelation in the process observations. Traditional authors have developed control charts that are based on plotting the residuals obtained from the forecasts of a time series model fitted to the original observations. Hence we propose a grey control chart which is designed by grey theory. This approach applies a novel algorithm and easy to monitor quality control. We presented a novel rolling grey EWMA (RG_EWMA) chart for process quality. Three contributions are proposed in our study: (1). this study can overcome the weakness of independent assumptions. (2). this research enables to modify data structure of a significant time series auto-correlated process observations. (3).our new model is sensitive and early detecting process changes with a small sample size.

1. Introduction

Control charts have been applied as a stable tool for detecting and upgrade the quality of production processes. Traditional SPC approaches define all process variables obey normal distribution behavior and independent. If the process unstable and failure, the process engineers have to monitor failure shifts as soon as possible. In this study, we explore the application of a novel rolling grey EWMA (RG_EWMA) chart model for improving quality control processes. The average run length (ARL) is selected to measure the effectiveness when a process shift is happened. The ARL denotes that an average number of observations are necessitated prior to an out-of-control situation is occurred. The benefit of our proposed grey approach is only required small samples to monitor the process changes. In the next section, the grey forecasting models are introduced and an overview of the proposed monitoring procedure will be given.

A traditional control chart assumed in process control is that the observations of the process are independent. Nevertheless, the independence assumption is not practical in real production processes, for instance continuous production processes. Several studies applied traditional control charts by modifying the control limits to account for autocorrelation when there is significant autocorrelation in the process observations. (Vasilopoulos and Stamboulis, 1978; Alwan and Roberts, 1988; Alwan, 1992; Maragah and Woodall, 1992; Padgett, Thombs, and Padgett, 1992; Yashchin, 1993; VanBrackle and Reynolds, 1997; Sheu and Lu, 2009).

Some approaches have designed new control charts by plotting the residuals attained from the forecasts of a time series model matched to the previous data. (Harris and Ross, 1991; Montgomery and Mastrangelo, 1991; Superville and Adams, 1994; Wardell, Moskowitz and Plante, 1994; Schmid, 1995; VanderWeil, 1996; Lu and Reynolds, 1999a; Sheu and Lu, 2008).

Facing today's competitive business environment and diversity in small production for customer's demand, the fact of statistical process control for small production is often challenged by insufficient information. Guo and Dunne (2006) proposed the grey predictive control chart based on the grey theory proposed by Deng (1982). The task of the grey control chart is a revelation of the whitening trend prediction of production process based on small observations. Guo (2006) investigated a small sample oriented grey interval-typed control chart for monitoring product quality. Ku and Huang (2006) explored the application of grey forecasting models for predicting and monitoring production processes and showed that the grey predictors are found to be superior to the sample mean when a mean shift occurs.

A number of different methods are proposed to improve the prediction accuracy of grey models. Rolling mechanism is adopted and revealed that the forecasting efficiency of rolling grey model was superior to GM(1,1) model. (kayacan et al., 2010; Hsu, 2011; Tang and Yin, 2012). In order to compare the residuals attained from forecasts of a time series model. In this paper, we design the rolling grey EWMA chart for production processes, which incorporates the merits of the grey theory and EWMA chart to improve the performance of the EWMA control chart. The rest of this paper is organized as follows. Section 2 describes GM(1,1) model; Section 3 presents a design of rolling grey chart and Section 4 experiments. Finally, some remarkable conclusions are drawn in Section 5.

2. GM (1, 1) MODEL

Many scholars have invented different forecasting methods over the years, in which the conventional ones include linear regression model, Markov method, time series model, ARIMA method and so on. (Chiu, 1988; Bianco et al., 2009; Hsu, 2011). However, these methods have limitations. Since such methods require a large number of samples while historical data collection is difficult. When there are less data for analysis and decision-making, The grey system theory is first put forward by scholar Deng (1982), who systematically models, forecasts and analyzes through a small number of incomplete information, in which the GM(1,1) model is widely used. Through the model establishment and simulation, the accuracy of the forecast model is analyzed and provided as a reference index for decision-makers. GM (1, 1) model construction process is described below:

Step 1: Assume original data sequence to be:

$$x^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\right\} \tag{1}$$

Step 2: A new series $x^{(1)}$ is generated by accumulated generating operation (AGO).

$$x^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \ldots, x^{(1)}(n)\right\} \tag{2}$$

where $x^{(1)}(1) = x^{(0)}(1)$ and $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$,

Step 3: Calculate background values $z^{(1)}$

$$z^{(1)}(k) = (1-\alpha)x^{(1)}(k-1) + \alpha x^{(1)}(k), \quad k = 2,3,\ldots,n \tag{3}$$

Step 4: Establish the grey differential equation.

$$\frac{dx^{(1)}(k)}{dt} + ax^{(1)}(k) = b \tag{4}$$

where a is the developing coefficient and b is the grey input.

Step 5: solve Eq.(4) by using the least squares method; then, the forecasting values can be obtained as:

$$\hat{x}^{(1)}(k) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-a(k-1)} + \frac{b}{a} \tag{5}$$

where $[a,b]^{\mathrm{T}} = \left(B^{\mathrm{T}}B\right)^{-1} B^{\mathrm{T}}Y$

$$Y = \left[x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\right]^{T}$$

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}$$

Step 6: The recovered data $\hat{x}^{(0)}(k)$ can be retrieved by the inverse accumulated generating operation (IAGO).

$$\hat{x}^{(0)}(k) = (1-e^{a})\left(x^{(0)}(1) - \frac{b}{a}\right)e^{-a(k-1)}, \quad k = 2,3,\ldots,n \tag{6}$$

3. Design of Rolling Grey Chart

Observations come from an autocorrelated process are considered in this study. Most applications, the ARMA(1,1) model can be convenient for acquiring original observations and estimating parameters. Hence, as long as parameters satisfy $0 \le \theta \le \varphi < 1$ and $\sigma_b^2 > 0$, the ARMA(1,1) model can be used to yield process observations. The ARMA(1,1) process can be represented as

$$X_t = (1-\varphi)\xi_0 + \varphi X_{t-1} + b_t - \theta b_{t-1} \tag{7}$$

where b_t is the random white noise of the ARMA(1,1) process at time t and follows independent normal random variables with mean 0 and variance σ_b^2, ξ_0 is the mean of the observation, θ is the MA parameter, and φ is the same AR parameter, respectively.

Rolling grey prediction model is a grey GM (1,1) model, which uses the same sequence in front of several data (usually before the start of a four-point) to establish GM (1,1) model, and then predict the next data (Fifth after the data), the value again after the shift point (fifth), the same build GM (1,1) model, so the original data so that until the last point, mainly to test the GM (1,1) of accuracy. This method is what we call rolling grey model detailed steps are as follows:

Step 1: List the sequence of autocorrelated observations

$$x^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\right) \tag{7}$$

where $x^{(0)}(k)$, $k = 1,2,3,\ldots,n$

Step 2: Examine rolling grey forecasting

Using the first 4 observations to build GM(1,1) forecasting model for the first time GM (1,1) model.

$$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), x^{(0)}(4))$$

$$x^{(1)} = (\sum_{k=1}^{1} x^{(0)}(k), \sum_{k=1}^{2} x^{(0)}(k), \sum_{k=1}^{3} x^{(0)}(k), \sum_{k=1}^{4} x^{(0)}(k))$$

Define grey differential equation $\frac{dx^{(1)}(k)}{dt} + ax^{(1)}(k) = b$, $x^{(1)}$ initial value $x^{(0)} = x^{(1)}$, and the general method for solving differential equations, discrete response obtained is:

$$\hat{x}^{(1)}(k) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-a(k-1)} + \frac{b}{a}, \quad k = 1,2,3,4 \tag{8}$$

IAGO recycling operations have the following formula

$$\hat{x}^{(0)}(k) = (1 - e^{a})\left(x^{(0)}(1) - \frac{b}{a}\right)e^{-a(k-1)}, \quad k = 2,3,4,5 \qquad (9)$$

Step 3: forecasting observations

Collect k = 5 be the predictive value of the fifth observation. Delete the oldest observation (the first data), and add the next observation (the sixth data) to rebuild GM(1,1) forecasting model in Step 2. The forecasting sequence can be found as

$$(\hat{x}^{(0)}(5), \hat{x}^{(0)}(6), \ldots, \hat{x}^{(0)}(n)) \qquad (10)$$

Step 4: residual data

Define the residual value is $\varepsilon^{(0)}(i) = x^{(0)}(i) - \hat{x}^{(0)}(i)$, $i = 5,6,\ldots,n$ and the residual sequence shows as

$$(\varepsilon^{(0)}(5), \varepsilon^{(0)}(6), \ldots, \varepsilon^{(0)}(n)) \qquad (11)$$

The residual sequence can be regarded as independent data with unknown mean and variance. This work utilizes the residual sequence to construct the rolling grey types charts for monitoring the mean of autocorrelated observations based on an ARMA(1,1) process. Figure 1 illustrates the different data pattern needs specific control chart to monitor the process shifts.

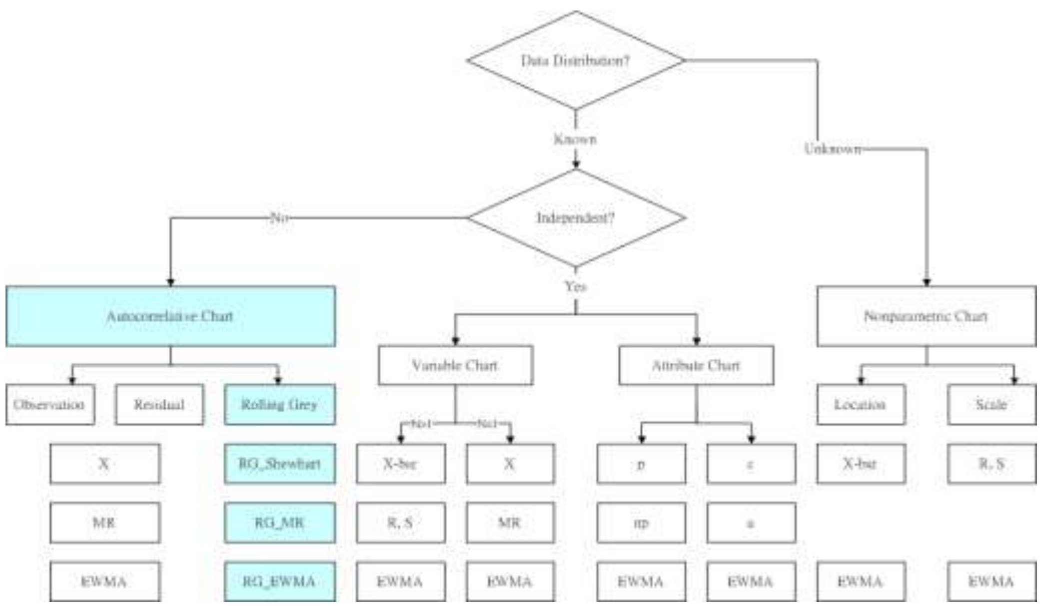

Figure 1. The classification of control charts.

The concept of control charts was developed by Shewhart (1920) and primarily to detect process mean shifts. Shewhart control charts are effective for detecting large changes in process parameters, but they are not sensitive to small changes in process parameters. Therefore, the cumulative sum (CUSUM) and EWMA control charts are continually developed to enhance the detection ability when the small process shifts. Besides, moving average (MA) charts plot the unweighted moving average over time for individual observations which are more effective than X-bar charts in detecting small process shifts. In particular, the charts perform usefully when there is only 1 observation per subgroup. However, EWMA charts are generally preferred over MA charts because they weight the observations. The attempt of this work is to develop rolling grey MA and EWMA control charts and compare the performance of them.

3.1 A Rolling Grey Shewhart Control Chart

The residual sequences (Eq.11) can be regarded as nearly independent control statistics to monitor the shift of a process. Supposing L denotes the width of the control limits, the central limit (CL), upper control limit (UCL) and lower control limit (LCL) of the rolling grey Shewhart (RG_Shewhart) chart can be written as

$$\begin{aligned} UCL &= \mu_0 + 3\sigma_0 \\ CL &= \mu_0 \\ LCL &= \mu_0 - 3\sigma_0 \end{aligned} \qquad (12)$$

3.2 A Rolling Grey Moving Range Control Chart

The residual sequences (Eq.11) can be regarded as nearly independent control statistics to monitor the shift of a process. Supposing L denotes the width of the control limits, the central limit (CL), upper control limit (UCL) and lower control limit (LCL) of the rolling grey moving range (RG_MR) chart can be written as

$$\begin{aligned} UCL &= D_4\overline{MR} \\ CL &= \overline{MR} \\ LCL &= D_3\overline{MR} \end{aligned} \qquad (13)$$

where $\overline{MR} = \sum_{i=1}^{n-1} MR_i \Big/ n-1$, the absolute of the two residual sequence is defined as $MR_{i-1} = \left|\varepsilon^{(0)}(i) - \varepsilon^{(0)}(i-1)\right|$, $i = 2,3,4\ldots,n$. When residuals, $\varepsilon^{(0)}(i) = x^{(0)}(i) - \hat{x}^{(0)}(i)$, falls outside the range of control limits, it indicates that the process is out of control and some actions should be taken adequately.

3.3 A Rolling Grey EWMA Control Chart

Utilize the residuals of rolling grey GM(1,1) model, we define rolling grey EWMA monitoring statistics

$$Z_t = \lambda\varepsilon^{(0)}(t) + (1-\lambda)Z_{t-1}, \; 0 < \lambda \le 1 \qquad (14)$$

where Z_t represents the t-th sequentially number of the process. Adopting the starting value, Z_0, is the target value of the mean μ_0. The mean and variance of $E(Z_t) = \mu_0$ and $Var(Z_t) = \frac{\lambda(1-(1-\lambda)^{2t})}{2-\lambda} \cdot \sigma_0^2$. If time is infinite then $Var(Z_t) = \frac{\lambda}{2-\lambda} \cdot \sigma_0^2$.

The control limits for the rolling grey EWMA (RG_EWMA) control chart are usually based on the asymptotic standard deviation of the control statistic. Hence the RG_EWMA control chart can be developed as follows:

$$UCL = \mu_0 + L\sigma_0\sqrt{\frac{\lambda}{2-\lambda}}$$

$$CL = \mu_0 \qquad (15) \qquad UCL = \mu_0 - L\sigma_0\sqrt{\frac{\lambda}{2-\lambda}}$$

Since the process has shifted, some adequate action should be taken whenever Z_t falls outside of the control limits are shown as Table 1

Table 1. The algorithmic description of Rolling Grey EWMA charts.

Input:
N : Rolling number
λ: the weight of EWMA scheme;
Output:

Out-of-control ARL_1 ;

1. Set the desired in-control $ARL_0 \approx 370$
2. Choose the rolling number
3. Generate pseudo Time-series original data sequence $x^{(0)}$
4. Adopt the first N data and execute the Grey forecasting for N+1 data
5. Calculate the residual $e(N+1) = x^{(0)}(N+1) - \hat{x}^{(0)}(N+1)$
6. Delete the oldest data and add the new one to execute the Step 4.
7. Collect the residual sequence $e(i)$
8: Calculate the Grey EWMA statistic z_t by Eq. (1)

Given appropriate value of L, the LCL and UCL can be calculated by Eqs. (3)-(5)

Count the Run Length when z_t exceeds LCL or UCL

9. Execute 100,000 iterations of the Steps 3-8, the ARL corresponding to the specific shift and (λ, L) combination is calculated.
10. Using the "Bi-Section" researching method, L corresponding to the desired $ARL_0 \approx 370$ is obtained through repeating Steps 3–9 under the process is in-control
11. Repeat Step 3-9 to compute ARL_1s under specific shift and Determine the minimal ARL_1s
12. Return optimal (λ^*, L^*)

4. EXPERIMENTS

In our experiments, we attempt to compare detection ability of the RG_Shewhart, RG_MR and RG_EWMA charts for monitoring small process shifts. This RG_EWMA chart will detect the quality level and feedback the signals of quality efficiencies and violation criterions. The quality performance index will be considered by the total balance performance of computing ARL_1 under specific shift and the algorithm description for minimal ARL_1 are shown as Table 1.

5. CONCLUSIONS

We propose a new approach that designs a new RG_EWMA chart which optimizes the quality of production process. In order to monitoring the quality efficiencies of the process criterions, we have found this RG_EWMA chart for detecting the small process shifts. We found that the RG_EWMA chart is not only more flexible than the RG_Shewhart scheme but also outperforms it. Moreover, the RG_EWMA chart is more efficient than the traditional chart for autocorrelated process observations. In future research, this approach can be applied and modified by varying all the key cost factors that affect the performance level of QoS in cloud chains.

References

Alwan LC. (1992) Effects of autocorrelation on control chart performance, Communication in Statistics: Theory and Method. 21: 1025-1049.

Alwan LC, Roberts HV. (1988) Time-Series modeling for statistical process control, Journal of Business and Economic Statistics. 6:87-95.

Chiu FL. (1998) Forecasting tourism: a combined approach, Tour. Manage. 515-520.

Bianco V, Manca O, Nardini S. (2009) Electricity consumption forecasting in Italy using linear regression models, Energy. 34: 1413-1421.

Deng JL. (1982) Grey system fundamental method, Huazhong University of Science and Technology, Wuhan, China.

Guo R. (2006) Grey quality envelop control charts, Journal of Quality. 13: 401-409.

Guo R, Dunne T. (2006) Grey predictive process control charts, Communications in Statistics - Theory and Methods. 35:1857-1868.

Harris TJ, Ross WH. (1991) Statistical process control procedures for correlated observations, The Canadian Journal of Chemical Engineering. 69: 48-57.

Hsu LC. (2011) Using improved grey forecasting models to forecast the output of opto-electronics industry, Expert System with Applications. 38: 13879-13885.

Hung KC, Chien CY, Wu JK, Hsu FY. (2009) Optimal alpha level setting in GM(1,1) model based on genetic algorithm, Journal of Grey System. 12:23-32.

Kayacan E, Ulutas B, Kaynak O. (2010) Grey system theory-based models in time series prediction, Expert System with Applications. 37:1784-1789.

Ku LL, Huang TC. (2006) Sequential monitoring of manufacturing processes: an application of grey forecasting models, International Journal of Advanced Manufacturing Technology. 27: 543-546.

Lu CW, Reynolds MRJr. (1999a) EWMA control charts for monitoring the mean of autocorrelated processes, Journal of Quality Technology. 31: 166-188.

Maragah HD, Woodall WH. (1992) The effect of autocorrelation on the retrospective X-Chart, Journal of Statistical Computation and Simulation. 40: 29-42.

Montgomery DC, Mastrangelo CM. (1991) Some statistical process control methods for autocorrelation data, Journal of Quality Technology. 23: 179-193.

Padgett CS, Thombs LA, Padgett WJ. (1992) On the α-risk for shewhart control charts, Communications in Statistics-Simulation and Computation. 21: 1125-1147.

Roberts SW. (1959) Control chart test based on geometric moving averages, Technometrics. 1:239-250.

Schmid W. (1995) On the run length of a shewhart chart for correlated data, Statistical Papers. 36: 111-130.

Sheu SH, Lu SL. (2008) Monitoring small shifts in the autocorrelated process mean and variance with GWMA charts based on residuals, Asia Pacific Journal of Operational Research. 25: 781-792.

Sheu SH, Lu SL. (2009) Monitoring the mean of autocorrelated observations with one generally weighted moving average control chart, Journal of Statistical Computation and Simulation. 79: 1393-1406.

Superville CR, Adams BM. (1994) An evaluation of Forecast-Based quality control schemes, Communications in Statistics-Simulation and Computation. 23: 645-661.

Tang HW, Yin MS. (2012) Forecasting performance of grey prediction for education expenditure and school enrollment, Economics of Education Review. 31: 452-462.

VanBrackle LN, Reynolds MRJr. (1997) EWMA and CUSUM control charts in the presence of correlation, Communications in Statistics-Simulation and Computation. 26: 979-1008.

VanderWeil SA. (1996) Modeling processes that wander using moving average models, Technometrics. 38: 139-151.

Vasilopoulos AV, Stamboulis AP. (1978) Modification of control chart limits in the presence of data correlation, Journal of Quality Technology. 10: 20-30.
Wardell DG, Moskowitz H, Plante RD. (1994) Control charts in the presence of data correlation, Management Science. 38: 1084-1105.
Yashchin E. (1993) Performance of CUSUM control schemes for serially correlated observation, Technometrics. 35: 37-52.