Analysis of sectoral performance in Borsa Istanbul: a game theoretic approach

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Keywords
Game Theory, minimax, sectoral portfolio, linear programming, Borsa Istanbul.

Abstract

Game theory has been the subject of many diversified fields of study in literature. Within game theory framework, portfolio optimization is one of the applications used in finance. The main goal of the study is to create optimal portfolios in Borsa Istanbul by using a game theoretic approach and to analyze relative performances of sectoral portfolios. In the study, portfolio optimization is set up as a zero sum-up game and converted to a linear optimization model. The stock market and investors are identified as players (opponents) in the model. Players have invested in the portfolios according to the minimum risk with maximum return criteria. The monthly data for the period between 2009 and 2014 is obtained from Borsa Istanbul. The data set includes the stock return of 229 companies and also includes returns of Borsa Istanbul four sector indexes: fiscal, industrial, service and technology. Relative performances of sectoral portfolios are evaluated by Sharpe Performance Index and Variation Coefficient. The main finding is that the model can be used in portfolio optimization. Technology-based portfolio attains the highest return with lowest portfolio concentration. Moreover, the relative performance of technology portfolio is higher than the other three sectors.

Introduction

In financial markets, the risk and uncertainty are universal matters. All investment decisions are shaped by risk and uncertainty. The risk comes from the obscurity of future. Inward and outward both economical and political challenges affect the markets. Risk basically inherits the nature of financial markets. As it is inevitable as a matter, the investors have to take it into account in decision-making.

The investing decisions are shaped by choosing from a vast array of financial instruments. Investors either invest in a single financial instrument or create a diversified portfolio that involves many assets. While investing more than one asset, the investor seeks to optimize the portfolio composition. In portfolio investment, the investor has to take two crucial decisions. One is to decide which financial instrument is included in the portfolio, and another is the weight of them. (Prigent, 2007). For that purpose, many models have been proposed in investment theory. The models differ in the perspective how they approach the optimization problem, and measure the risk. Some models accept the minimum variance as a measure of risk while some other uses semi variance, beta coefficient, variation coefficient or maximum loss.

Modern portfolio theory was put forth by Harry Markowitz (1952) by his publishing ''Portfolio Selection''. In his study, Markowitz developed a quadratic programming model. The model is based on Mean-Variance Model, andthe risk is identified as variance. It defines the investors as risk averse, keen to optimize the portfolio by maximizing the expected return for a given level of risk. In case the investor takes on higher risk, the expected return must be higher. The risk aversion of investors determines how much risk to take on. Hence, all the investors hold their portfolios as subject to their risk perception.

In Mean-Variance Model, the problem appears in working with large asset universe.In case, the asset universe is large, the estimation of covariances become quite difficult. In order to overcome the difficulty in calculation, William Sharpe (1963) developed Single Index Model and then following other index models. The model measures the interrelation between the mean return of market and stocks instead of correlations among the stocks. The measure of relation is defined as Beta Coefficient by Sharpe, and it can be expressed by simple linear regression.

William Sharpe (1971) remarked that if the portfolio analysis problem was appropriate to apply linear programming techniques, the success of application would be substantially enhanced. Linear programming has found a wide usage in investment decision models in the literature. Martin R.Young (1998) was the first applied Minimax Theorem to the portfolio selection problem. Young's model solve the portfolio selection problem by linear programming. The assumptions of the model are based on game theory. Young's portfolio selection model, expressed Minimax Model aims to minimize the maximum loss for a given level of return. The risk is identified as the minimum return contrary to the variance models. In the model, the portfolio maximizes the minimum return or, alternatively, minimizes the maximum loss.

Minimax Theorem was introduced by John von Neumann in 1928. The theorem is basically used in decision making under uncertainty. Game theory was then accepted as a discipline in 1944 with "The Theory of Games and Economic Behavior" by John von Neumann and Oscar Morgenstern. Game theory formulates the strategies of opponents mathematically, in the decision-making process.

In game theory, games are played under risk and uncertainty. From this point of view, financial markets and games are similar in all their aspects. The portfolio optimization problem can be built up two-person zero-sum games. In the game, the loss of one player equals the gain of the other and so the sum of the game is zero. The market and the investor might be identified as opponents. Each behaves rationally and chooses the best strategy for their own. The game fundamentally is a single- player game that is played against Nature. (Shubik, 1989) .The market represents the Nature with all its characteristics. (Friedman, 1997). It has numerous strategies that are affected by various economic, political and social turmoil's like the volatility of exchange rates and interest rates. In consideration of the complexity of the factors, we cannot solely determine the effect of each factor on the overall market. The assumption is valid for the stock exchange market as well. The change in asset values also reflect the economical, political and social changes on the market, butanone of them is solely responsible for this change.

In the game, the opponents choose their strategies in order to maximize the expected minimum returns (Maximin criterion) or to minimize the maximum expected loss (Minimax criterion).In other words, the market seek to maintain the minimum loss and the investor as an opponent, choose the best alternative among the worst outcome with minimum risk. Hence, the model presents a quite conservative approach.

In literature, Papahristodoulou and Dotzauer (2004) compare the Mean-Variance model (MV), Mean Absolute Deviation Model (MAD) and Minimax model (MM)in Stockholm Stock Exchange between the periods 1997 and 2000. They note that Maximin Model is most robust according to other models. Cai et al. (2004) compare the Minimax with Markowitz's quadratic programming models in terms of risk in Hong Kong Stock Exchange. They find that the results of both models has similar performance. Nevertheless, Minimax is not sensitive to the data. Farias et al. (2007) compare MV Model, MM Model and MAD Model in Brazilian Stock Market (BOVESPA). They point out that Minimax Model is superior to others. Hassan et al. (2012) produce the asset allocation by using Maximin Model between the crisis years 1997 and 2001 in Malaysia.

They remark that the model provide consistent result in crisis period. Schaar Schmidt and Schanbacher (2014) analyze the effectiveness of Minimax Model in portfolio allocation between the period 1990 and 2010. The analysis reveal that the portfolio weights are stable over time in Minimax Model and model performs better in terms of risk approach.

The analysis in this paper can contribute to the literature as follows: The study aims to examine an alternative approach for game theoretic models. In order to measure the effectiveness of the model, the analysis focuses on four sectoral portfolios. We can divide the analysis into three phases. In the first phase, the portfolios are produced based on Minimax approach by which the weights for sectoral portfolios is obtained. In the second phase, the portfolio return is calculated according to the average return of stocks during the analysis period. Following that, the risk of portfolio is computed by the covariances between the selected stocks.

In the third phase, the relative performances of the four portfolios are evaluated by using Sharpe Performance Index and Correlation Coefficient. At the end of the analysis, the performance of portfolios is compared with indices values. We note that the model might provide a useful and efficient approach for the investment decision and an alternative model for portfolio selection.

Data and Methodology

The aim of research to obtain the stock allocation of portfolio generated via Minimax approach in Borsa Istanbul to assess the relative performance of four sectoral portfolios and to determine the superior portfolio to all other.

Data

The data consists of 229 stock prices and sectoral indices values. The data set involves four compilations of data. Each data compilation contains the stocks trading in main sectors of Borsa Istanbul. The main sectors used in the analysis are fiscal, industrial, service and technology. Besides the portfolios, the sectoral indices are computed. All the data are obtained from Borsa Istanbul. Transaction costs and taxes are ignored in the study. The analysis is run between the period January 2009 and December 2014. Monthly closing prices are used to assess the change in stock values. The intermittent stocks are excluded from analysis. The risk-free rate is taken from the website of the Central Bank of the Republic of Turkey. The rate is computed as the average of compounded government bond yields as observed within the analysis period.

The indices values are used as a measure in comparison with portfolio returns. All simulations are run via Microsoft Excel Software.

Methodology

The payoff matrix is generated according to the returns of stock. The years are displayed on the matrix rows (market strategies), and the stocks are on columns (investor strategies).Each cell in the matrix represents a payoff value.The stocks can be held in the portfolio for a minimum period of 1 month. Initially, the log returns of stocks are calculated as following:

$$RM_t = \frac{R_t - R_{t-1}}{R_{t-1}}$$

where

RM_t Is the monthly return of the stock

R_t is the stock price at the time(month) t

R_{t-1} is the stock price at the time(month) t-1

The payoff matrixes are constructed for 12 months. The matrixes were built for the four sectoral portfolios separately. It is almost impossible to know which strategy the market will choose. However, we might assign probabilities, say S,to each stock. In a zero-sum up game, the outcome of the game can be achieved by linear programming. The model is used only to decide which stock is included the portfolio and what is the weight of the stock in the portfolio.

The model is transformed to linear programming. The technology indices are composed of 12 stocks and the formulation for technology sector is expressed as follows:

$$\sum_{i=1}^{12} a_{ij} S_i \geq V,\ j = 1,2,3,4,5$$

where

$i$is the strategy of the investor

j is the strategy of themarket

a_{ij} is the pay off for an investor with strategy$i$ when the market strategy is$j$

S_i is the probability of strategy$i$ (investment share of strategy i)

V is the value of the game

The expected returns are calculated for each month in the same way. The probabilities sum up to one.

$$\sum_{i=1}^{12} S_i = 1$$

Both sides of equations are divided by$V$, as defined by equation:

$$P_i = \frac{S_i}{V}$$

After the transformation, the equation is described as

$$\sum_{i=1}^{12} a_{ij} P_i \geq 1$$

The aim of investor is maximizing the gain (V) . We can approach the aim in another side by minimizing 1/V. In this sense, the objective function is as follows:

$$MinZ = Min\frac{1}{V} = \sum_{i=1}^{12} P_i$$

All probabilities have to be positive as well.

$$P_i \geq 0$$

When all simulations were solved with linear programming model, the stock allocation for sectoral portfolios was acquired.

	Stock Code	Company Name	Portfolio Allocation
1	ALCTL	Alcatel Lucent Teletas	0.002
2	ANELT	Anel Telecom	0.156
3	ARENA	Arena Computer	0.145
4	ARMDA	Armada Computer	0.224
5	ASELS	Aselsan	0.128
6	DGATE	Datagate Computer	0.113
7	ESCOM	Escort Technology	0.009
8	INDES	Indeks Computer	0.095
9	KAREL	Karel Electronic	0.035
10	LOGO	Logo Software	0.052
11	NETAS	Netas Telecom	0.042

Table 1: Portfolio allocation for technology sector

In the second phase, we calculated risk and return of portfolios. The portfolio return with historical data will be calculated as follows:

$$R_p = \sum_{i=1}^{N} w_i.R_i$$

where

R_p is the average return of the portfolio

w_i is the weight of stock*i* in theportfolio

R_i is the average return of stock*i*

The risk of the portfolio is computed with variance. The square root of variance gives the standard deviation of the portfolio. Standard deviation denotes the risk associated with an asset or portfolio.

$$\sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j Cov_{ij}$$

where

σ_p^2 is the variance of the portfolio

σ_i^2 is the variance of stock*i*

Cov_{ij} is the covariance between stock*i* and stock*j*

The variance of each stocks is not solely a measure for overall portfolio risk, the covariance matrix was built for each sectoral portfolios in the analysis.

Technology Sector	
Standard Deviation	0.0769
Portfolio Return	0.0305
Number of Stocks in Portfolio	11
Fiscal Sector	
Standard Deviation	0.0728
Portfolio Return	0.0277
Number of Stocks in Portfolio	28
Service Sector	
Standard Deviation	0.0773
Portfolio Return	0.0239
Number of Stocks in Portfolio	24
Industrial Sector	
Standard Deviation	0.0764
Portfolio Return	0.0274
Number of Stocks in Portfolio	25

Table 2: Risk and return of sectoral portfolios

From Table 3, we can deduce that the return of technology-based portfolio provided the highest return to the investor in the period 2009 and 2014.

Since the risk is critically important for the decision maker, especially for the investor with risk-averse characteristics, the risks were also compared portfolios. The fiscal-based portfolio carries the lowest risk with the standard deviation of 0, 0728.The difference between the highest (service sector) and lowest values (fiscal sector) is roughly 6% in portfolios. In general, the risks are quite close between the sectors.

On the other hand, the concentration of portfolios was evaluated relatively. Despite the stock universe was considerably large for industrial sector, optimal portfolio composition was achieved by 25 stocks. The fiscal-based portfolio has the greatest number of stocks, and the technology-based portfolio contains 11 stocks with the minimum number among all. That means the optimal portfolio composition was achieved with less stock than other portfolios during that period.

	Technology		**Fiscal**		**Service**		**Industrial**	
	Portfolio	Indice	Portfolio	Indice	Portfolio	Indice	Portfolio	Indice
Return	0.0305	0.0342	0.0277	0.0207	0.0239	0.0181	0.0274	0.0241
Risk	0.0769	0.0897	0.0728	0.0911	0.0773	0.0558	0.0764	0.0638

Table 3: Comparison between sectoral portfolios and indices

Indices are commonly used as a benchmark in the analysis. Hence, portfolio returns and risks are compared with relative indices in Table 4.In comparison, three portfolios except technology attained higher return than the indice. Since each portfolio has different return and risk, we need a common measure to examine the relative performances of portfolios.

In the third phase, the relative performances of four portfolios are evaluated by Sharpe Performance Index and Correlation Coefficient. In the analysis, both are used as benchmark.

In portfolio management, the performance of a portfolio is a crucial matter. As the performance presents the success of portfolio, it helps the investor to decide whether the portfolio will be held for future.

William Sharpe (1994) developed an index to assess the performance of a portfolio. The performance index is built as the excess return of the portfolio over the risk-free rate, divided by the portfolio risk.

The index is a risk- adjusted measure of returns. Sharpe used three variables in assessing the performance of portfolio: the return of a risk-free asset, the average return of an asset and the standard deviation of return. It is commonly used in risk/return measures. Sharpe performance index is formulated as follows:

$$SharpePerformanceIndex = \frac{R_p - R_f}{\sigma_p}$$

Where

R_p is the average return of the portfolio

R_f is the risk-free rate

σ_p is the standard deviation of the portfolio

The higher the index, the better the performance of the portfolio. Variation Coefficient is a ratio of the average return to the standard deviation. It is a useful tool to the investor in the evaluation of alternatives with different expected return.

$$VariationCoefficient = \frac{\sigma_p}{R_p}$$

In general, the lower the variation coefficient, the better the performance of a portfolio.

Results and Considerations

The sectoral stocks in Borsa Istanbul are used in our analysis. The reason for using a sectoral analysis is to guide the investor about which sector brings gain more than others.

For a better understanding of the performance results, we can express the differences between the return of sectoral indices with portfolio returns in percentage terms. The aim of the comparison of indices is to comprehend what if the investor prefers investing the indices instead of holding portfolio.

If the indices were chosen for investment, the investor recorded loss for three sectors: industrial, fiscal and service. The sectoral index that provides gains the investor more than generated portfoliois only the technology one. If the investment was made to the technology indices in the period 2009 and 2014, the investor's gain was approximately 12% higher than portfolio return.

On the other hand, the gain of technology index accompanies with high volatility. Also, an outstanding inference might be done for the service sector. The service indices was brought almost 41% loss for the investor. As a sequence, we can deduce that the gain of indices except technology indices is lower than generated portfolios.

Sectoral Portfolios	Variation Coefficient	Sharpe Performance Index
Technology Sector	2.519	0.237
Fiscal Sector	2.624	0.212
Industrial Sector	2.785	0.198
Service Sector	3.231	0.150

Table 4: Performance of sectoral portfolios

Table 4represents the relative performances of generated sector-based portfolios. The portfolios are ranked from the lowest variation coefficient to the highest. According to the results, technology-based portfolio outperforms with the lowest variation coefficient and the highest Sharpe Performance Index.

The superior portfolio is generated by the stocks trading in technology sector. In recent years, the technology sector has been growing steadily in Turkey as well as in the World, and shares of technology companies offer high returns to the investors in stock market. As a consequence, the results sustain that the method is efficient in portfolio optimization that allocates the convenient stocks for portfolio diversification. For future studies, the game theoretic model can be examined with other portfolio optimization models in the aim of a benchmark. Furthermore, an analysis might be conducted with sub-
Sectors, and that will provide noteworthy results.

Conclusions

In this study, we aim to build up a comparative analysis of sect oral portfolios between the years 2009 to 2014 in Borsa Istanbul and to guide the investors for future investment.

Also, we attain to build portfolios by a game theoretic model. The analysis is run in three steps. At first, we search for optimal portfolio allocation in fiscal, industrial, service and technology sectors according to the Minimax approach in decision-making.
We then construct portfolios with the minimum risk and maximum return. The game theoretic model achieves to build up optimal portfolios. The return and risks of generated portfolios are compared with each other, and also with the indice. At the third step, the relative performance of sectors is compared with Sharpe Performance Index and Variation Coefficient.

Since the analysis is run on historical data, the approach has a deterministic structure like most investment analysis models. In the portfolio, the performances of the stocks, as well as their weights, might vary over time. However, past performances would be a helpful for the investors about forthcoming investment decisions.

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