Modeling of the economic-political systems

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Abstract
This article presents the general economic system, the main purpose of which is to meet all the basic needs of society. Important features of the economic system are described. The classification of the economic system according to the ability to meet the main goal is given. It also takes into account the fact that the essence of the economic system depends on the political system in society, which is constant at certain intervals of time, but can change at any time unknown in advance.

Indicators of efficiency and effectiveness of the economic system are introduced, which give us information about the state of the economic system at a given time interval and at a given point in time, respectively. Based on the performed analysis, three types of tasks for optimal management of the economic system are posed. One simple optimal control example is presented for illustration.

Introduction

Economic-political system is unity of possibility complex actions, which provides all the basic public service needs satisfaction in every time moment from a given time interval. Society's production is mainly involved in the governance of the existing material and labor resources. In addition, the economic system depends on the essence of the current political system, which is constant at the given time interval. All of them received from the general economic-political system. Obviously, the economic-political system has a cyclical nature and is conjugate with the time interval (cycle). Therefore, it provides information about the weaknesses of the economic-political system only at the end of the cycle.

The model of the general economic-political system is discussed. We generate a specific regressive equation of the economic-political system, which envisages different conditions of the political system. Based on the generated regressive equation, we give an appropriate analysis.

Statement of the problem

An economic system is a set of tools that work together to satisfy all the basic needs of society at any given time interval.

An economic system is optimal if it satisfies all the basic needs of society at minimal cost. Denote the sum of the basic needs of the society at a given time t from the interval T by $D(t)$:

$D(t) = \{D_1(t), \ldots, D_n(t)\}$,

Where $D_i(t), (i = 1,2, \cdots, n)$ is the community's i-th basic demand at t moment, and n is the number of basic needs at t moment, which generally varies with time: $n = n(t)$.

Denote the starting moment of the time interval (i.e., the cycle) with t_0 and the last moment with t_1 i.e., $T = [t_0, t_1]$. Let $K(t)$ be the set of community-owned material resources at the moment t i.e.

$K(t) = \{K_1(t), \cdots, K_k(t)\}$,

Where $K_i(t), (i = 1,2, \cdots, k)$ is an i-th material resource, expressed in the corresponding units, and k is their total number. Let $L(t)$ be the set of community-owned labor resources at the moment t i.e.

$L(t) = \{L_1(t), \cdots, L_k(t)\}$

Where $L_i(t), (i = 1,2, \cdots, k)$ is an i-th labor resource, expressed in the corresponding units, and k is their total number.

Thus, the state of the economic system at each time moment t will depend on the following variables: the initial t_0 moment; The length of the time interval T, i.e., at t_1 moment; At the basic needs unity of society every moment of time, i.e. At $D(t)$; At set of community-owned material resources every moment of time, i.e. At (t) ; At the set of community-owned labor resources every moment of time, i.e. At $L(t)$ and at investment every moment of time, denote by $I(t)$, $I(t) = \{I_1(t), \cdots, I_k(t)\}$, where $I_i(t), (i = 1,2, \cdots, k)$ is the amount of investment made in the i-th field at time t. Denote the condition of the economic system of society at any given time moment by $E(t)$. Obviously, $E(t)$ as an endogenous variable will depend on: t_0 and t_1 numerical parameters and $D(t)$, $K(t)$, $L(t)$, $I(t)$ vector exogenous variables. I.e., We can write that

$E = E(t_0, t_1, D(t), K(t), L(t), I(t))$. The private face of the function $E(t)$ can be a production function:

$$Y(t) = AK_1^{\alpha_1} \cdot K_2^{\alpha_2} \cdot \dots \cdot K_n^{\alpha_n} \cdot L_1^{\beta_1} \cdot L_2^{\beta_2} \cdot \dots \cdot L_n^{\beta_n} \cdot I_1^{\gamma_1} \cdot I_2^{\gamma_2} \cdot \dots \cdot I_n^{\gamma_n}$$

To evaluate the effectiveness of the functioning of the economic system, we need to take into account eE- the rate of effectiveness of the economic system. At the given time moment t an indicator of satisfying i-th basic needs is

$$s_i(t) = D_i(t) - Y_i(t),$$

Where $Y_i(t), (i = 1,2, \cdots, n)$ is the volume of output produced for satisfying i-th demand at the moment t. If $s_i(t) > 0$ - the demand is satisfied overestimation at the t - moment. If $s_i(t) = 0$ - the demand is satisfied exactly at the t – moment. If $s_i(t) < 0$ - the demand is fully not satisfied at the t - moment. Everywhere below implied that every value, except time, is represented in monetary units.

Definition 1. The effectiveness rate of the economic system at the time interval $T : t_0 \le t \le t_1$ is

$$eE(T) = \int_{t_0}^{t_1} \sum_{i=1}^{n} s_i(t)dt$$

Definition 2. An economic system is optimal over a T time interval if the following conditions are fulfilled:

$$s_i(t) \ge 0, \forall\, i = 1,2, \cdots, n\, ; t \in T.$$

Thus, an optimal economic system over a T time interval is able to satisfy all the basic needs at any moment from this time interval.

Quasi-optimal economic systems are usually more widespread, not optimal economic systems that sometimes cannot be implemented.

Definition 3. An economic system is quasi-optimal over a time interval T if the following conditions are fulfilled:

$$\sum_{i=1}^{n} s_i(t) \ge 0, \qquad \forall\, t \in T\,.$$

It is clear that if the economic system is optimal, it is also quasi-optimal. The opposite provision is unfair because in the case of a quasi-optimal economic system, some of the indicators of basic needs $s_i(t)$ can be negative (i.e., some requirements may not be fully satisfied). In this case, to fulfill the condition of definition 3.

$$\sum_{i=1}^{n} s_i(t) \ge 0, \qquad \forall\, t \in T\,.$$

The sum of the basic needs at the time moment t must be non-negative, which means that at the moment t the surplus output produced by some of the basic needs and that total product must overlap (at its value) the product needed to satisfy other basic needs. If the above condition is violated at some time interval, but so that the following condition remains in effect:

$$eE(T) = \int_{t_0}^{t_1} \sum_{i=1}^{n} s_i(t)dt \geq 0$$

then such an economic system is integrally quasi-optimal. I.e., The deficit arising from various basic needs is covered by excess production not instantaneously, that is, every t moment, but throughout the whole T interval. Such an economic system may present difficulties if the time interval T is large. If an integral condition is also violated, then the economic system is unsatisfactory - it can in no way satisfy all the basic requirements of the T -interval. Thus, we got the following:

Statement of the first model.

The economic system can potentially satisfy all the essential requirements over the time interval T and only when the efficiency rate of this system is non-negative over the time interval T, i.e.

$eE(T) \geq 0.$

The following naturally follows:

Definition 4. The economic system at time T interval is called:

extremely satisfactory or effective if $eE(T) > 0$; Satisfactory or Neutral if $eE(T) = 0$; Unsatisfactory i.e., ineffective if $eE(T) < 0.$ It is clear that the optimal (as well as quasi-optimal) system can be both effective and neutral. If we compare these economic systems together, we get: Rational is a neutral optimal economic system - it satisfies all basic needs at all times of the time interval T and does not create excess products (i.e., rational use of material and labor resources). Ideally is an effective optimal economic system - it satisfies all the basic needs at every time moment of time interval T and generates a surplus product in advance. Reliable is an effective quasi-optimal economic system, because at the end of the cycle (i.e., for time moment t_1) it generates a reserve (excess product) that contributes to the reliability of the next cycle.

Risky is a neutral quasi-optimal economic system, since it contains a risk factor over a time interval T and does not create a reserve at the end of the cycle (i.e., for time moment t_1), which enhances the risk factor over the next cycle. Unacceptable is an ineffective economic system as it cannot satisfies basic needs over a time interval T.

From the Definition 1 It follows directly that the effectiveness rate of an economic system is of a cyclical nature and is bound by the time interval T. It therefore provides information on the economic system's performance only at the end of the cycle and does not provide any information on the functioning of the economic system at any time moment t from the time interval T. To get this information we need to introduce a new function of time called the economic system efficiency indicator.

Definition 5. Efficiency indicator of an economic system we call a function given by the following integral

$$X(t) = \int_{t_0}^{t} \sum_{i=1}^{n} s_i(\tau)d\tau \ , \qquad t \in T \ .$$

From this definition it follows that

$eE(T) = X(t_1),$

I.e., The efficiency indicator of the economic system over the time interval $T:\ t_0 \leq t \leq t_1$ coincides with the value of the effectiveness rate at the end of the cycle, i.e., at the moment t_1. The efficiency indicator of the economic system gives information about the system at any time interval $t_0 \leq t \leq t$ $(t \leq t_1)$. For example, if $X(t) > 0$, the economic system is overly satisfying, i.e. it is effective at time interval $[t_0, t]$; If $X(t) = 0$, then the economic system is satisfactory or neutral at time interval $[t_0, t]$; If $X(t) < 0$, the economic system is unsatisfactory or ineffective at time interval $[t_0, t]$. The economic system of some nature at the time interval $t' \leq t \leq t''$ may have a tendency to change its nature. To get this information, we need to introduce efficiency gradient indicator of the economic system.

Definition 6. The gradient indicator of efficiency of an economic system at any moment t from time interval T is

$$\frac{dX(t)}{dt} = \sum_{i=1}^{n} s_i(t), \qquad t_0 \leq t \leq t_1 . \qquad (1)$$

Statement of the second model

The value of the gradient indicator of the efficiency of the economic system at any time moment from the interval T provides the following information:

If $\frac{dX(t)}{dt} > 0$, then the economic system has a positive tendency; If $\frac{dX(t)}{dt} = 0$, then the economic system has a tendency to change its nature i.e., has a neutral nature tendency; If $\frac{dX(t)}{dt} < 0$, then the economic system has a tendency of a negative nature. From the formula (1) we directly have

$$\frac{dX(t)}{dt} = u(t), \quad t_0 \leq t \leq t_1 , \qquad (2)$$

Were

$$u(t) = \sum_{i=1}^{n} s_i(t) = \sum_{i=1}^{n} Y_i(t) - \sum_{i=1}^{n} D_i(t) = \sum_{i=1}^{n} (Y_i(t) - D_i(t)), \quad t_0 \leq t \leq t_1 , \qquad (3)$$

is the indicator of all major's aggregate demand and its real aggregate satisfactory at any time moment t from the time interval T.

The first optimal control problem.

Consider the control process of an economic system over the time interval $T:\ t_0 \leq t \leq t_1$ in a generalized form. We have:

$$(I) \qquad \frac{dX(t)}{dt} = u(t), \qquad t_0 \leq t \leq t_1;$$

$$(II) \qquad X(t_0) = X_0\ ,\ \ X(t_1) = X_1 \qquad ;$$

$$(III) \qquad u \leq u(t) \leq U, \qquad t_0 \leq t \leq t_1;$$

Here X_0, X_1, t_0, u, U are given numbers. We need to find the control $u(t): t_0 \leq t \leq t_1$, that satisfies (I), (II), (III) conditions and the time $t_1 - t_0$ will be a minimum, i.e. We want the given $X(t_0) = X_0$ efficiency indicator of the economic system (which determines the nature of the economic system) through permissible control (as shown in (III)) reduced to the efficiency indicator $X(t_1) = X_1$ in a minimal time ($t_1 - t_0$ is minimal). For example, make the economic system of a negative nature ($X_0 < 0$) a neutral or positive nature economic system ($X_1 \geq 0$) in the shortest possible time.

We want to change the given $X(t_0) = X_0$ efficiency of the economic system (which determines the state of the economic system) through permissible management $u \leq u(t) \leq U, t_0 \leq t \leq t_1$, in the shortest possible time and transfer to the given efficiency $X(t_1) = X_1$, which determines the another economic state. For example, switch from neutral (first) to effective (second) or vice versa.

This is a simplest time optimal control problem with fastened ends. If we apply the principle of maximum, we will have either $u(t) = u$ or $u(t) = U$ over the entire $t_0 \leq t \leq t_1$ interval. In the first case we have

$$X_1 = X_0 + (t_1 - t_0)u , \qquad (4)$$

And in the second case we have

$X_1 = X_0 + (t_1 - t_0)U$. (5)

Obviously, if there is a solution of the considering optimal control problem (for given X_0, X_1, t_0, u, U values) and $X_1 > X_0$, then the optimal control and optimal time will be $u(t) = U,\ t_0 \le t \le t_1$, and $t_1 - t_0 = \frac{X_1 - X_0}{U}$, respectively.

If $X_1 < X_0$, then optimal control and optimal time will be

$u(t) = u,\ t_0 \le t \le t_1$, and $t_1 - t_0 = \frac{X_0 - X_1}{u}$, respectively.

Remark 1. Since $u(t) = \sum_{i=1}^{n}\big(D_i(t) - Y_i(t)\big),\quad t_0 \le t \le t_1$, and $D_i(t), (\ i = 1,2,\cdots,n.)$ are already given values (requirements), so we are controlling the process through production volumes i.e., by $Y_i(t), (\ i = 1,2,\cdots,n.)$ production functions.

Remark 2. For theoretical considerations, if $Y_i(t), (\ i = 1,2,\cdots,n.)$ are advanced given values (are production volumes), then we are controlling the process through demand volumes i.e., by $D_i(t), (\ i = 1,2,\cdots,n.)$ demand functions.

Remark 3. We do not exclude cases where some components of control are production volumes and others are requirements.

Consider a qualitative factor such as a political system or, more simply, a political regime. Note such a factor with the letter P. We assume that the political system can be of three types: the first, the transitional, and the second. In this case, the qualitative variable P in the econometric model (regression equation) is as follows:

$$P_1 = \begin{cases} 1, & \text{if the system is in transition state,} \\ 0, & \text{if the system is in other state;} \end{cases}$$

$$P_2 = \begin{cases} 1, & \text{if the system is in second state,} \\ 0, & \text{if the system is in other state.} \end{cases}$$

If we take into account fictitious variable $P_1(t), P_2(t)$ denoting the political system, then the state of the economic system of society at any given moment of time, referred to as $E(t)$, takes the form as follows:

$E_p = E\big(t_0, t_1, D(t), K(t), L(t), I(t), P_1(t), P_2(t)\big)$.

The private face of this function can be the production function:

$Y(t) = AK_1^{\alpha_1} \cdot K_2^{\alpha_2} \cdot \dots \cdot K_n^{\alpha_n} \cdot L_1^{\beta_1} \cdot L_2^{\beta_2} \cdot \dots \cdot L_n^{\beta_n} \cdot I_1^{\gamma_1} \cdot I_2^{\gamma_2} \cdot \dots \cdot I_n^{\gamma_n} \cdot B_1^{P_1} \cdot B_2^{P_2}$,

Where B_1 and B_2 are unknown parameters, and P_1 and P_2 are fictitious variables.

Take into consideration the change in the political system, the optimal control problem set out above (I), (II), (III) will take the form of a variable-structure optimal control problem. We call this problem the second optimal control problem.

6.The second optimal control problem.

Consider optimal control process of economic system, at $T:\ t_0 \le t \le t_1$ time interval, in such a general form:

$$\begin{cases} \dfrac{dX(t)}{dt} = u_1(t), & t_0 \le t \le \theta_1, \quad (7) \\ \dfrac{d\tilde{X}(t)}{dt} = u_2(t), & \theta_1 \le t \le \theta_2, \quad (8) \\ \dfrac{d\hat{X}(t)}{dt} = u_3(t), & \theta_2 \le t \le t_1, \quad (9) \end{cases}$$

$X(t_0) = X_0,\quad X(\theta_1) = \tilde{X}(\theta_1),\quad \tilde{X}(\theta_2) = \hat{X}(\theta_2),\quad X(t_1) = X_1$;

$$
\begin{aligned}
&u_1 \le u_1(t) \le U_1, &&t_0 \le t \le \theta_1;\\
&u_2 \le u_2(t) \le U_2, &&\theta_1 \le t \le \theta_2;\\
&u_3 \le u_3(t) \le U_3, &&\theta_2 \le t \le t_1.
\end{aligned}
$$

Where $X_0, X_1, t_0, u_1, U_1, u_2, U_2, u_3, U_3$ are given numbers; $\theta_1, \theta_2, t_1 \in T$ are unknown moments from a given time interval.

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