Using the real options lattice to select and plan capital projects

Mark Smith
LeTourneau University, Houston, TX, USA

Warren Matthews
Belhaven University, Houston, TX, USA

Robert Driver
LeTourneau University, Houston, TX, USA

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Abstract
Go with the cash flow. Real options methods determine value of a proposed project. In some ways, real options resemble the common net present value method; the net of expected future cash flows is discounted to the present (in today's currency). For example, one would select projects with a projected net present value (NPV) greater than zero, when future cash flows exceed expenditures. When proposed projects are mutually exclusive, the highest NPV determines the winner. However, real options expand the traditional NPV formula and open up a wider range of choices. The tools of real options analysis include both discrete and continuous stochastic analysis, partial differential equations, Black-Scholes modeling, and decision trees. Of these methods, the lattice-type tree offers the simplest means to value and explain real options. This paper will demonstrate real options analysis using the recombining binomial lattice, and present a sample project scenario. (This is the original work of the authors and not previously published.)

1. Introduction

For those less familiar with discounted cash flow procedures, the following brief review is offered in this Introduction, along with a basic explanation of real options. Real options may be regarded as a special kind of project decision-making method. Project decision making using real options is generally undertaken from the financial perspective(Ragozzino, Reuer, and Trigeorgis 2016). Real options analysis should perform best when certain conditions hold true. These include the following (Mun 2016):

- Management makes rational strategic decisions based on accurate financial information.
- Management has the flexibility to act or change course.
- The flexible strategy may be practically applied.
- Uncertainty creates the need for a planned strategy.
- Uncertainty impacts the project's value.

Real options depart from traditional strategic methods, both in the valuation of resources and in strategic decision making. But first, consider the traditional NPV model of valuation.

The traditional NPV formula derives from the concept of time value of money, which in finding present value means discounted cash flows (DCF).A discount rate is applied to a future expected cash flow to compute present value, or the value of an asset in today's currency. For example, the face value of a thousand-dollar, zero-coupon bond that matures in five years may be discounted annually at a given discount rate for the bond's remaining life. Supposing this discount rate to be 4%, then

$$BondValue = PV_b = M \text{ x } \frac{1}{(1+0.04)^5} = \$1,000 \text{ x } 0.8219 = \$821.90.$$

The above calculation assigns a current value to the zero-coupon bond.

In this calculation, PV_b represents the present value (in today's dollars) of the bond's expected future cash flow. M represents the face value, or future value, of the bond at maturity (five years hence), while $\frac{1}{(1+0.04)^5}$, also stated as $(1.04)^{-5}$, represents the present value discount factor. Tables of the PV discount factor often use the term "present value interest factor" (PVIF). Such a factor is used to discount from some future expected cash flow to the present; therefore, the discount rate in this case would be 0.04.

The value of an investment, such as a revenue-generating project, may be calculated by fundamental analysis, using the net present value of all future, expected cash flows. Now NPV includes both the present value of such positive cash flows from the above example, as well as any negative cash flows, such as the expenditure of resources. Traditional calculations, for simplicity's sake, use a negative cash flow for the initial outlay of resources, or $-CF_0$. So

$$NPV_{TRADITIONAL} = \sum_{t=1}^{n} CF_t - CF_0.$$

In this calculation, the capital sigma (∑) represents the sum of the positive annual cash flows designated by year t, from $t = 1$ to $t = n$, the last year of the project. The cash flows are assumed to occur at the end of each year. As mentioned, the initial outlay is given by $-CF_0$. Assuming the project life is five years, with the positive cash flows occurring at the end of each year, the annual discount rate is 4%, and the initial outlay occurs on the first day of Year 1, the equation is modified as follows:

$$NPV_{TRADITIONAL} = (CF_1 \text{ x } 1.04^{-1}) + (CF_2 \text{ x } 1.04^{-2}) + (CF_3 \text{ x } 1.04^{-3}) + (CF_4 \text{ x } 1.04^{-4}) + (CF_5 \text{ x } 1.04^{-5}) - CF_0,$$

in which $-CF_0$ means the initial expenditure and CF_1 through CF_5 represent each year's positive cash flows. (For the sake of simplicity, this equation assumes no terminal cash flow.)

2. Real Options: the Preliminary Example

Now real options consider an alternate set of cash flows that contribute to $NPV_{TRADITIONAL}$, modifying the NPV formula by adding the value of the real options, or NPV_{REAL}. Therefore, the strategic NPV appears as follows:

$NPV_{STRATEGIC} = NPV_{TRADITIONAL} + NPV_{REAL}$ (Gitman and Zutter 2015, p. 437).

The following preliminary, real-options example begins from the above equations, $NPV_{TRADITIONAL}$ and $NPV_{STRATEGIC}$.

Suppose the above example is continued by stating the initial outlay and the annual cash flows as follows:

$CF_0 = -\$1,000,000.$

$CF_1 = \$200,000.$

$CF_2 = \$400,000.$

$CF_3 = \$0.$

$CF_4 = \$200,000.$

$CF_5 = \$200,000.$

Using the same discount rate and other assumptions, the traditional NPV would be

$$NPV_{TRADITIONAL} =$$
$$(\$200,000 \text{ x } 1.04^{-1}) + (\$400,000 \text{ x } 1.04^{-2}) + (\$0 \text{ x } 1.04^{-3}) + (\$200,000 \text{ x } 1.04^{-4})$$
$$+(\$200,000 \text{ x } 1.04^{-5}) - \$1,000,000$$
$$= \$192,300 + \$369,840 + \$0 + \$170,960 + \$164,380 - \$1,000,000$$
$$= -\$102,520.$$

Under the traditional NPV valuation, the project is not acceptable with this negative net cash flow. Notice that Year 3 shows a cash flow of zero. If one were to propose that Year 3 involve downtime, in which project resources-in-waiting could be allocated to generate alternate revenue, the real option could be added under the strategic NPV. One possible alternate application might be placing project resources under lease contract during the downtime in Year 3. The hypothetical company decides on a flexibility option to lease its project resources for a year at a net charge of $150,000.At the end of Year 3, the project comes back on line. Then $NPV_{STRATEGIC} = -\$102,520 + (\$150,000 \text{ x } 1.04^{-3}) = \$30,829.$ The NPV is now positive; therefore, under $NPV_{STRATEGIC}$, the project becomes acceptable.

Three major assumptions took place in the above scenario. First, only one discount rate applied. The second assumption pertains to the type of real option. There are in fact many real options that may be available in a project that involves many changing risks, exogenous and endogenous factors, and industry-related and market-related opportunities.

The third assumption considers the fact that real options are, in fact, optional. Furthermore, in some respects they resemble financial options. For instance, a real option gives the management the right, but not the obligation, to pursue the option, usually within a given timeframe. In fact, the American form of call or put option is similar to such decision strategies. For example, the decision to lease the project resources during downtime could possibly be arranged to occur at any convenient time during the project lifetime and exercised at management's discretion. This real option would occur at a decision node on the proposed binomial lattice, and would be valued based on projected cash flows.

3. Discussion of Risk and Real Options Methods

The three traditional project decision-making approaches include the market approach, the income approach, and the cost approach. The income approach is favored herein for two reasons. First, income or revenue may be easily related to the firm's financial statements such that the project may generate the pro forma financial statements. Second, the use of net positive revenue with the income approach is readily applicable to discounted cash flows.

Real options methods undertake valuation of resources as a cost-benefit scenario with DCF as the foundation. However, the simple method of NPV, which in the example in Section 2 seemed rather basic, assumes an immutable consideration of factors that, in reality, are dynamic in nature. In fact, the variation of risk between benefits and costs would suggest that cash flows could be discounted using at least two discount rates—one rate for revenues, and one for costs. Long-term project decision-making may struggle to obtain a single, ideal discount rate, if such a rate could be accurately predicted.

As the project duration extends longer—such as beyond three years—real options should be considered. A flexible real options strategy could adjust the discount rate with the new options as sub-projects or replacement projects.

The volatility of the project is important when considering range of value from a simulated cone of uncertainty, along with project intervals when decisions may be reached. On a lattice-type tree, such decision points are called nodes.

Along with volatility and time intervals marked by decision nodes, this paper will also consider up-and-down jumps in value based on a discrete accounting for Brownian motion. The lattice method will therefore imply a discrete simulation of Brownian motion, which draws in this case from random walk theory—the concept that value is not predictable from expected returns (fundamental analysis) or historical patterns (technical analysis). However, the first step in the process does consider the traditional DCF to derive a first-pass NPV, a form of fundamental analysis (Reddy and Clinton, 2016).

In the big picture, there are three phases of the analysis. Keep these steps in mind in the study that follows:

(1) Calculating the DCF to derive the first-pass estimate of net present value for future expected cash flows over the life of the project. This is the traditional, static NPV that is generally understood by management.

(2) Deriving the first lattice by using the NPV in Step 1 to project into the future the underlying asset's nodes. This is the lattice of the underlying asset, also simply called the underlying, which calculates each node from left to right(Ragozzino, Reuer, and Trigeorgis 2015).

(3) Deriving the second lattice—the lattice of the option—by backward induction. This means moving from right to left, determining the terminal nodes first to derive the intermediate and beginning nodes (Mun 2016).

For the lattice of the underlying, values jump either up or down at each node, with the lattice resembling a cone of uncertainty used in continuous stochastic analysis. This lattice will be simpler than the continuous analysis, but still approximate the values found in more difficult mathematics. In fact, one major advantage of the lattice-type tree is that discrete intervals and the basic algebra are much more palatable for management to utilize. At the same time, the analyst may use sophisticated software to verify the results from the lattice method. High-powered continuous evaluation methods will not be explained in detail in this study. However, in the limit, the lattice values approach the values of sophisticated, continuous models. The sample analysis in this paper will consider five discrete time intervals for the project life, representing an annualized discount rate and time-steps. But these time-steps may be extended to up to thousands of discrete time intervals by using analytical software. In reality, the time-steps could be semiannual, quarterly, monthly, or daily.

With regard to risk, it should be noted that forecasting is an inexact science with some very precise mathematical concepts and algorithms. (Consider target practice as an analogy. A rifle may be precisely inaccurate, producing a tight pattern that misses the bull's eye.) In spite of the best-laid plans of finely-tuned minds, the mathematics cannot perfectly predict the unpredictable, even using statistical analysis and stochastics, the math of uncertainty. In a previous paper, the authors indicated that outliers occur in derivative forecasting, as made abundantly apparent by large-scale financial disasters at Long Term Capital Management, Lehman Brothers, and other firms that lost their footing (Smith, Matthews, and Driver 2015,p. 318). The human errors of directional trading and failure to allow for the rare exceptions to the norm (outliers) can cause a financial failure. Financial options are risky by their nature, as are business projects, but at some point the firm must still make a decision to commit financially.

4. What Are the Real Options?

The following list of real options touches on a few of the more common decision nodes in project planning (Mun 2016, and Gitman and Zutter2015,p. 437):

- Option to abandon the project.
- Option to expand the project.
- Option to contract (downsize at savings).
- Option to choose among option strategies.
- Compound option (value of option depends on value of another option).

- Option to change strikes (implementation costs).
- Option to change the volatility.
- Sequential compound option.
- Timing option.
- Flexibility option (e.g., making a contract or agreement on resources)

Each type of option has its own approach to the lattice method. Some of them simulate a call or put option. The following example of an option to expand resembles an American call option in the financial market.

5. The Binomial Lattice and a Real Option to Expand

Assume that the revenue approach is used for project decision. This example will choose expansion as the real option. Step 1 will be assumed as complete, which calculates the net present value of annualized cash flows over five years. Steps 2 and 3 will be illustrated as the "Lattice of the Underlying" and the "Lattice to Value the Option," respectively.

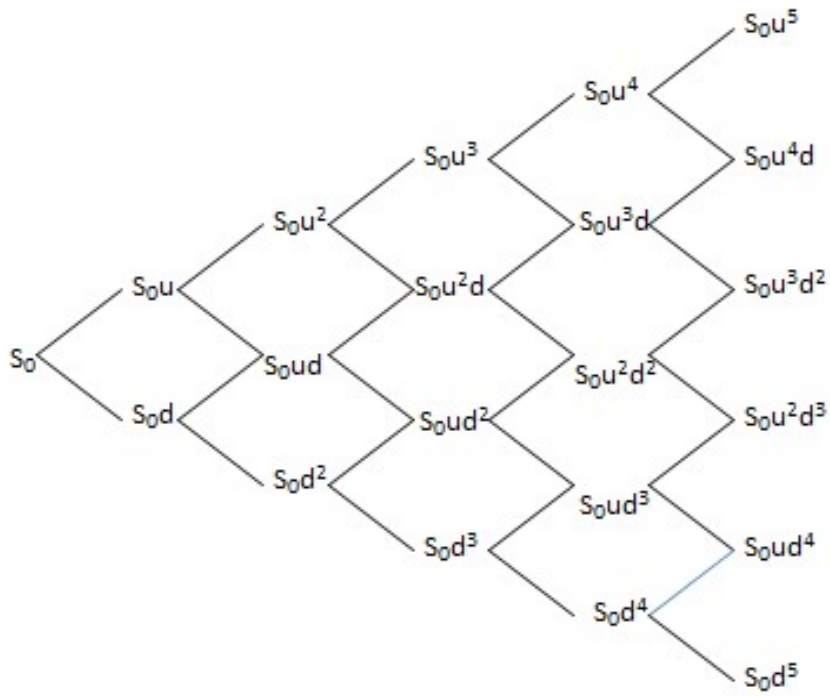

Figure 1: Lattice of the Underlying, Generic

The binomial lattice in common use involves two branches forward in time (to the right) for each node, such that nodes conjoin from two previous nodes that are vertically adjacent. This is called a recombining binomial lattice. (Another construction would develop a lattice where recombining does not occur, which is not studied in this paper.) The generic, recombining binomial lattice at five simple steps would look like Figure 1 (Kijima 2013, and Mun 2016).

The present value of the underlying, S_0, is derived from the expected cash flows taken from the discounted cash flows annualized over five years(Step 1), with S_0 as the project's expected value at Day One. Each node, where the text in the lattice indicates S_0 times variations of up or down jumps labeled u and d, respectively, represents a potential decision point at the end of an annual period. For example, at the end of Year One, an increase in value by one jump is shown as S_0u .Jumps in value, either up or down, are attributed to random walk theory. The formulas for jumps in value would be $u = \exp(\sigma\sqrt{\delta t})$ and $d = \exp(-\sigma\sqrt{\delta t})$, where

u is the upward jump factor,
d is the downward jump factor, which equals 1/u,
e is the banker's constant, expressed above as exp raised to the power of $\sigma\sqrt{\delta t}$.
σ is volatility, and
δt is the incremental change of time between steps. For example, with an annualized cash flow and a five-year project lifetime, δt would be 1/5, or 0.2.

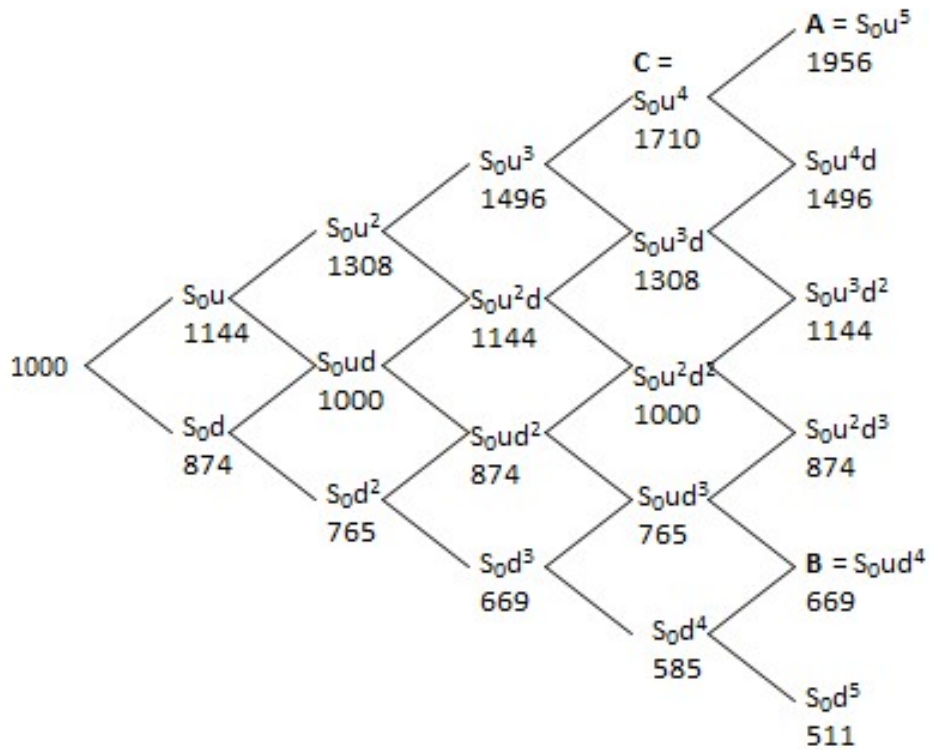

Figure 2: Step 2, Lattice of the Underlying in Dollars (000s)

In the lattice approach, one may change variables such as the discount rate (used in Step 1, the DCF that finds the first-pass present value). One may change other variables along the way. The only variable that usually does not change is the calculated volatility. (Volatility estimating and the potential for change of volatility will be discussed presently.) For this example, assume the volatility, σ, to be 30%, based on the historical logarithmic returns. Then $u = \exp(0.3\sqrt{0.2}) = 1.14358$, and $d = \exp(-0.3\sqrt{0.2}) = 0.87445$.

Now consider a calculated present value from Step 1, the DCF, of $1,000. The lattice of the underlying would look like Figure 2.

The variation in value from the original DCF calculation of present value, shown on the extreme left in Step 2 as $1,000, depends on a constant volatility and proportionate jumps up and down, as shown by the predictable pattern of the equivalent nodes on the same horizontal level. The range of future values (five years hence) goes from $1,956 to $511. If volatility were zero and there were a 100% chance that the DCF values would be accurate, the lattice would collapse to a linear value of $1,000 for all periods. Since volatility is greater than zero and each probability is not 100%, the lattice takes the shape of a triangle. So the lattice consists of a discrete simulation of the continuous cone of uncertainty. In fact, the greater the quantity of nodes over the same project

lifespan, the more the granularity and the closer the estimated values come to the continuous stochastic model.

Note also that the Lattice of the Underlying predicts future value, going from left to right. Now the analyst shall value the option to expand the project based on favorable conditions. Any lattice of an option will comprise Step 3, going from right to left, thereby deriving today's value of the option from the future, terminal values of the underlying. This process is called backward induction, and is determined by an equation of risk-neutral probability, $NodeValue = [(p)up + (1-p)down]\exp[-Rf(\delta t)]$ which is derived from the previous lattice of underlying node values, as well as the banker's constant (e, or exp to the power in brackets in the next equation), the risk-free rate of return (Rf), and the time factor (δt). To derive probability, p, enter

$$p = \frac{\exp[Rf(\delta t)] - d}{u - d}.$$

The risk-free rate applies frequently to financial valuation and is generally assumed to be the rate of return on a three-month U.S. Treasury Bill. In this calculation, assume Rf to be 5%. This assumes the probability to be risk-neutral by using the discount rate only in the DCF of Step 1, so here one would apply only the risk-free rate to avoid double counting. The time factor, δt, continues as 0.2 on the five-year project life, or one year out of five. The upward jump factor, u, continues as 1.14358; the downward jump factor, d, continues as 0.87445. Then

$$p = \frac{\exp[Rf(\delta t)] - d}{u - d} = \frac{\exp[0.05(0.2)] - 0.87445}{1.14358 - 0.87445} = \frac{0.13560}{0.26913} = 0.50385$$

,where p is the probability of occurrence of the upward jump, u.

Since there are two possibilities per node, up and down, and the sum of all probabilities is one, then $p + (1-p) = 1$. So the formula for Node Value weights the values of probabilities times the upward and the downward values as $(p)up + (1-p)down$. The Lattice for the Option to Expand takes on the values using the Node Value formula, which is repeated here: $NodeValue = [(p)up + (1-p)down]\exp[-Rf(\delta t)]$. For each node, the value derives from the next right-hand nodes on the options lattice.

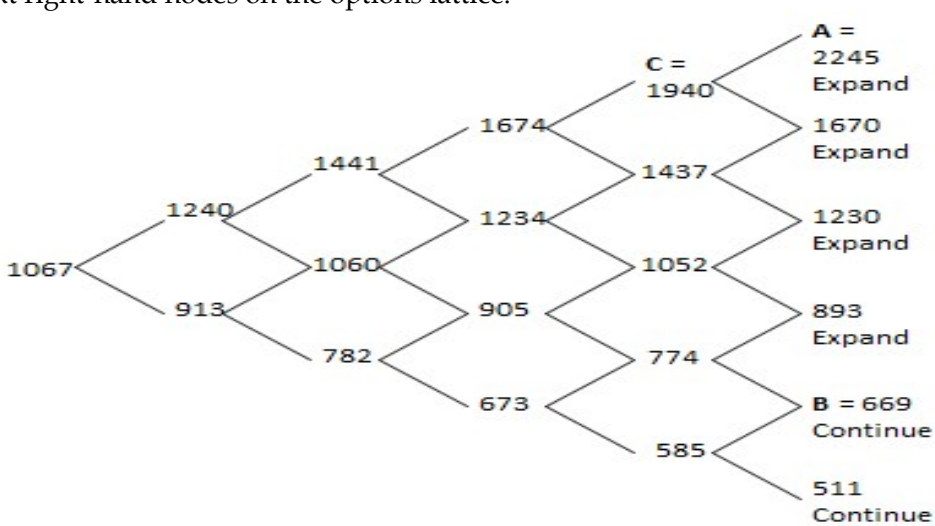

Figure 3: Step 3, Lattice to Value the Option in Dollars (000s)

Now consider the option to expand the project, beginning with the terminal nodes. Assume that the firm has the option to expand by acquiring the operations of another company at a cost of $200. Assume the acquisition would increase its cash flow by 25%. For simplicity, assume also that

the firm chooses to acquire the new operations at the end of year 5. Use the same terminal node from the lattice of the underlying, labeled A. Therefore, a 25% increase in the $1,956, less the $200 acquisition cost, would be $2,245 on the terminal node A of the Lattice to Value the Option.

Now the firm has the option to expand or not, and will choose according to profit maximization. Node A on the Underlying Lattice is compared to the corresponding node on the Options Lattice. Next, continue to compare each terminal node. Notice that node B on the Options Lattice is labeled Continue. This signals that the equivalent node on the Underlying Lattice is greater than the option to expand, so the firm chooses to continue operations "as is." $669 \text{ x } 1.25 - 200 = 636 < 669$. Instead of indicating $636 on node B, $669 carries over node B from the Underlying Lattice to the Options Lattice.

The rest of the Options Lattice is developed with backward induction from the terminal node values. Node C is calculated as follows:

$Node\ Value\ C = [(0.50385)up + (1 - 0.50385)down]\exp[-0.05(0.2)$,where $up = 2,245$ and $down = 1,670$.

So $Node\ Value\ C = (1131.14 + 828.57)(0.99005) = 1,940$. The remainder of the nodes on the Options Lattice are left for the reader to verify.

In summary, Steps 2 and 3 utilize these equations: $u = \exp(\sigma\sqrt{\delta t})$, while $d = \exp(-\sigma\sqrt{\delta t})$, and $p = \dfrac{\exp[Rf(\delta \text{t})] - d}{u - d}$, in which up and down jump factors and a risk-neutral probability equation are used to value the underlying and the option's node probability, respectively. Once p is determined, the nodes on the Options Lattice are further valued by $NodeValue = [(p)up + (1-p)down]\exp[-Rf(\delta t)]$.

6. Trinomial and Multinomial Lattices

In addition to the binomial lattice method, other methods may include the trinomial lattice or the multinomial lattice. These lattices are much more complicated and involve many calculations in the software. However, certain projects may be structured more appropriately for these alternate lattices. For example, the basic structure of the trinomial lattice would appear as indicated in Figure 4 (Mun 2016, and Kijima 2013).

The recombining trinomial lattice assumes three choices per node, which takes the firm in a different direction for the determination of upward and downward jumps, as well as probabilities. For simplicity, the above example shows only three time periods. The upward jump at three choices per node is $u = \exp(\sigma\sqrt{3\delta t})$. The downward jump is $d = \exp(-\sigma\sqrt{3\delta t})$. The implementation of the recombining trinomial lattice and other more complicated methods could be the subject of further study.

7. Volatility

Normally in the lattice method, volatility is the constant. Volatility may be estimated from several methods. The historical logarithmic returns from cash flows provide a sound approach to volatility. A popular method from historical logarithmic returns would apply this formula: $Volatility = \sigma = \sqrt{\dfrac{1}{n-1}\sum_{i=1}^{n}(x_i - X)^2}$,

where x_i is each natural logarithm derived from its corresponding return, X is the average of the logarithmic returns, and n is the number of returns by period. The volatility, σ, is expressed as a percentage. For example, the first month may have a cash flow of $100, while the second cash flow

may be $111. The return would be $\frac{\$111}{\$100} = 1.11$. Then x_i would be the natural logarithm derived from this return. That is, $x_i = \ln 1.11 = 0.104$. Assume $n = 5$ and five logarithmic returns are 0.104, 0.225, 0.325, –0.100, and –0.345. Then the average logarithmic return is $X = .042$. By the volatility formula, $\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - X)^2} = \sqrt{0.25(0.28735)} = 0.268 = 26.8\%$.

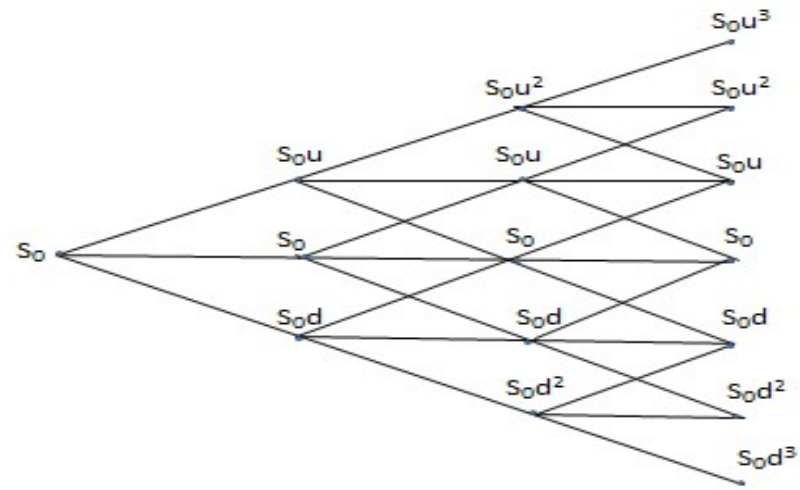

Figure 4: Recombining Trinomial Lattice Structure

More advanced methods, such as the Monte Carlo simulation, may be used to estimate volatility more reliably, especially when occasional negative cash flows are produced from returns. (The natural logarithm of a negative amount does not exist.) This leaves the chance for the historical returns to yield a volatility figure that could be less accurate than desired. Other methods of volatility estimation could supplement the simpler volatility formula cited above (Mun2016).

With regard to geometric Brownian motion, the volatility estimate may be the biggest weakness of the lattice method (Charnes 2007,pp. 190-191).This may be compensated by comparing the historical logarithmic estimates with results from more sophisticated methods such as the aforementioned Monte Carlo simulation. As mentioned earlier, volatility remains the most consistent variable, and may practically be treated as a constant once it is calculated with confidence. However, there may be exceptions. For instance, a change in overall parameters of the project could require a new estimate of volatility. Therefore, there is an option for change in volatility(Mun 2016).

8. Conclusions

Ideal answers to project decision making remain elusive and not normative. However, the use of real options with an appropriate method should help management to proceed with more confidence. Real options using the recombining binomial lattice employ a three-step valuation to make a project decision. These include fundamental analysis, using DCF to find the NPV of the underlying; application of random walk theory with a discrete simulation of Brownian motion; and backward induction to value the option nodes using a risk-neutral equation. Analysts who have had difficulty conveying complicated valuation methods to upper management should find the lattice method a more palatable means to explain the valuation of both the underlying and the options. The simpler, discrete decision nodes should promote a confident flexibility in responding to market conditions.

9. Research Limitations and Suggestions for Further Research

This study is an introduction to the binomial lattice model with one scenario oaf real option to expand. Other real options and models may be explored in similar works on the subject. (Mun 2016, Trigeorgis 1996, and Kijima 2013).

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