Implied volatility based approach to asset pricing

Robert G. Kuklik &Vladislav Vacek
University of Economics in Prague, Czech Republic

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Abstract
The aim of this paper is to propose an alternative approach to classic asset pricing. This is done by altering perception of risk from ex-post calculated parameter which is expected to remain constant in the future to ex-ante implied volatility derived from market expectations. The proposed model is constructed by combining mentioned ex-ante component with ex-post component obtained empirically by econometric methods. The resulting model is discussed and tested on selected market data.

1. Introduction

Motto:

„…a non - degenerate market equilibrium will arise only when there are sufficient profit opportunities, that is, inefficiencies, to compensate investors for the costs of trading and information gathering“. (A.W. Lo, 2007)

Development of the modern integrated theory of investment management which would be analytical as well as normative in its approach, has relatively a very brief history dating from the early ´50s of the last century. Until then in the developed Western economies, the „traditional“ approach was rather descriptive than analytical. With the grave experience of the Great Depression of the ´30s, the study of finance focused more on only liquidity, debt - austerity questions and the defensive aspects of financial survival such as protectionist tariff policies, etc.

In the 1950s´a great interest arouse in capital budgeting with development of new methods and techniques for selecting capital investment opportunities with the aim to achieve efficient capital allocation in the economic system. Various valuation models were developed for use in the financial decision - making and with the concern for valuation processes. In the ´50s, however, emerged a critical appraisal of the capital structure and the role of the firm´s dividend policy in relation to its valuation as a whole and therefore of its securities.

The breakthrough articles of Modigliani and Miller in 1958 and 1961 with their assumptions of „perfect“ financial markets free of imperfections([3]), where the debt or dividends did not matter from the point of view of securities´ valuation, and the Samuelson´s ideas in 1965 concerning the capital market efficiency, set a path for extended research enquiry which in fact continues today. A following significant event in the ´60s was the definition and development of portfolio theory and its eventual application to financial decision - making, first expounded by Markowitz in 1952 and subsequently refined and extended by Sharpe, Lintner, Mossin and Fama ([4], [12]) as well as other researchers. The process of diversification of risk in the portfolio situation as a marginal contribution to the overall portfolio risk being the function of a degree of correlation of the security return with the other securities´returns was therefore introduced.

The attempt to refine the Markowitz´s concept of the total risk was in the ´70s the Sharpe´s outline of the Capital Asset Pricing Model based on emergence of the Efficient Market Hypothesis framework ([4]) outlining the efficient market as a trading environment where the security prices reflect a maximum information available to the investors. Apart from the notion of the risk associated with an individual firm, this concept introduced the new categories of the „market risk“, „risk- free“ return, both forming the expected return in an efficient market and therefore the era of coefficient β has started. The oversimplified assumptions underlying the EMH have been under the circumstances of the actual market performance gradually proved less and less realistic. For example, the new field of inquiry called „behavioural economics“ (see [5]) shows that investors are not always rational, alike, and also self - interested individuals thinking in terms of some theoretical utility function when having „homogeneous expectations". Their decisions can certainly become distorted by several other factors as for example emotions, the „herd effect",etc., leading then to mis-assessment of the relevant probabilities.

The market processes are in reality very complex where for example, as Mandelbrot pointed out ([8]), the assumption that price changes follow a „Brownian motion" in the framework of the Random Walk Model is rather controversial in the light of contemporary facts. The periodic incidence of financial crises has finally prompted a view that pricing deviations could be in fact a natural part of the capital market´s development, where its smooth, „efficient" performance surface is only a special case, with more or less a random incidence of normal distribution of the price return values. Discontinuity of price trends, far from being an anomaly to be ignored, is in fact an essential ingredient of market's performance. The Mandelbrot´s framework based on volatility - the Fractal Dimension and serial dependence of returns reflected by the Hurst Exponent (see [9]), both in concert describing the market behaviour, have inspired an alternative approach to market pricing of the risky assets as outlined in the model being the key topic of this article.

2. The Volatility Asset Pricing Model (VAPM)

2.1 General framework outline

The model was developed with the idea that the market price performance *discontinuity* is a normal, natural phenomenon reflecting the element of volatile functioning of the physical world, which in conjunction with the price series *concentration,* i.e. having small changes in one price caused by small changes in another price „smoothly related", are the major ingredients in a realistic evaluation of the risk/return relationship. To put it another way, the long- term market memory, in other words the serial price dependents e.g. reflected for example in the Hurst Exponent (the Mandelbrot´s *„Joseph Effect"*) and parameter, characterising volatility expressed as Fractal Dimension by entropic process which encapsulates diversity of a hypothetical data population space (the same author´s *„Noah´s Effect"*), are the conceptual foundations of this model.

The basic paradigm is based on assumption that the investor´s risk/return assessment is the product of comprehensive expectations concerning the *total risk.* In other words, such approach is conceptually *synthetic,* not for example considering any classical components such as risk - free rate, risk premium, systematic and unsystematic risk. Investor's complex behaviour can be expressed as combination of historical experience (corresponds to above-mentioned market memory) and ex-ante expectations, in our case, quantified with implied volatility.

$$R(j,p) \;=\; \{\eta\,(j) \;+\; \varepsilon\,(j,p)\} \;+\; \rho(j)\,\Delta\,VIX(t) \;\; + [\textstyle\sum_k k\;\varphi(j,k)\;R(m,p-k)] \qquad (1)$$

for:

$p = (t + 30\ days)$ and $k = 1 \dots n$, and security j

where:

$R(j,p)$. . .is the dependent variable representing the security j / market index return, in time p days, i.e. the percentageprice change $P(j,p) / P(j,t)$, determined on the daily moving basis;

$\eta(j)$. . . is the expected constant component of the corresponding return $R(j,p)$, representing continuity as an average effect of the past returns;

$\varepsilon\ (j,t)$. . .is a random dispersion of the expected past return's continuity $\eta(j)$;

ΔVIX. . .is the independent variable as a 30 - day percentage change of the VIX Index in time$t$reflecting the volatility of the S&P 500 market index in the period 30 days hence, i.e. $\Delta VIX(t) = VIX(t) / VIX\ (t\text{-}30)$;

$\rho\ (j)$. . . is the sensitivity coefficient of the expected return $R\ (j,p)$ to a change in the market volatility expectations (i.e.$\Delta\ VIX$);

$R(m,p - k)$ . . .is the market index S&P 500, or a selected security´s 30 day return, lagged in the specified past period on the daily moving basis;

$\varphi(j,\ k)$ . . . is the sensitivity coefficient of the expected return $R(j,p)$ on the lagged returns $R(m,\ p - k)$ in the past $n = (p - k)$ periods; the complementary variable applicable to the *extended model* as an expression of variable serial dependence;

2.2Methodology

The *VIX Index* selected to express the model´s volatility component, also known as "indicator of fear" has generally an inverse relationship to the underlying market index S&P 500. In other words, when the market index is rising index VIX is typically falling and vice versa, and it is expressed as a point deviation from the expected market index average thus capturing its envisaged volatility. The VIX' construction uses near-term and next - term out - of the money SPX options with at least 8 days left to expiration and then weights them to yield a constant, 30 day measure of the expected volatility of the S&P Index. It is therefore an *expectation* type of indicator as opposed to a typical *ex post* indicator such as the market index itself.

We used VIX because it is well standardized and available for testing. But we must realize that this choice limits the model, as it's formulated above, to S&P 500 index. For general usage VIX must be replaced with other index of similar construction based on options of the security in question. The construction of VIX is also the reason for (t + 30 days) construction above. Because VIX(t) corresponds to expectation of value *R(S&P500, t+30)* andVIX(*t-30*) to *R(S&P500,t)*. The current price change form P(t+30)/P(t) was determined empirically as the best form of difference (linear and logarithmic differences were also tested). The *expected risk profile* can be in the framework of this model expressed as:

$$\rho(j)^2\,\sigma^2(VIX) \;+\; \sum_k[\varphi(j,k)^2\,\sigma^2\,R(m,p-k)] \;+\; \eta^2(j)\,\sigma^2\,\varepsilon\,(j)) \qquad (2)$$

for$k$ = *1...n*

Where the first terms reflects a dispersion of the expected volatility index being followed by the ex post dispersion of the lagged market index, and the third term features the random dispersion of the expected constant component of the continuity of the past return's serial dependence. The model´s expected return is therefore a function of the anticipated volatility of the market performance ρ and the non- volatility-induced part representing the expected constant continuity of the past performance η, together with the continuity lagged variable part φ.

As far as the portfolio selection is concerned, because the notion of the "risk - free" return or a "residual risk" as such is not considered, the classical portfolio selection cannot be based only on the so called "excess return" consistent with for example the CAPM and SIM frameworks, but on the optimizing the appropriate correlation matrix which reflects the total risk of individual securities assembled in the portfolio in the Markowitz´s sense. On the other hand, when the return on the whole market performance is considered, the process of selection would need to take place across several security markets, in order for the index aggregate variance to be optimized.

To test the model outlined above, the simple/multiple linear regression routine was applied, where, in the first instance the changes in the volatility index VIX and also the lagged performance of the market index were selected as the independent variables. The regression coefficient ρ then represents a percentage degree of responsiveness of the actual return's volatility, i.e. of the market index S&P500 as the *dependent variable*, to a percentage change in volatility expectations expressed by the VIX Index, as the *independent variable*, thus measuring sensitivity of the market index return to such expectations.

This latter variable was further complemented by an additional *independent variable* φ expressing the changing sensitivity of the expected return *R(j)* on the past performance of the market index/security, thus representing the *"variable market memory"* factor. Component η of the return *R(j)* ,with its random dispersion term ε, is interpreted as a constant proportion of the *non-volatility-induced effect* on such a return, i.e. *expected size of the serial dependence*, given it isrepresenting the inertial influence of the past index´ performance then complementing the effect of additionalvariable φ.

The outlined view of course contrasts with the usual interpretation of the coefficients a and β related to the "standard" models, e.g. the SIM where the perception of the risk is typically divided into the a-based, "non-market-induced" factors with its corresponding residual fluctuations ε, and the β-based factor to be strictly a reflection of the so called "market - associated" risk component. Generally, it is assumed that using various information segments, the market participants form their investment attitudes when deciding on the expected market performance then expressed by the combination of call/put option values placed on the market index, whilst also considering the past interacting trend of security prices and their returns. The associated risk is therefore viewed as a single category based on these two factors.

The *Coefficient of Determination* R^2 used in this model as an evaluation yardstick, reflects an extent to which the volatility expectations concerning the market and the lagged factors determine the actual ex post performance outcome of either the market or a security return. To assess a significance of the complementary auto correlated lags influencing the corresponding returns, the regression residuals were tested using the *Box - Jenkins method.* All values of residuals' autocorrelation and partial autocorrelation (up to 12 lags) functions remained within the 95% confidence level interval, thus proved to be near zero with only four single peaks of no significant pattern.

Further to that, the *Durbin - Watson Statistics* was derived in order to assess, in conjunction with the *Standard Error of Estimate,* the significance of residual terms concerning the serial dependence. Also the short - term prediction accuracy of the model was appraised by using computation of the *Mean Absolute Percentage Estimated Error (EER)* of the following month forecast. In addition, for the sampled companies as a matter of comparison, the "classical" *Single Index Model* was applied producing the corresponding coefficients a and β for the selected periods in the standard fashion. In this context, considering the VAPM framework, the coefficients $(\eta + \varepsilon)$, ρ and φ representing the market price structure, ie. the volatility as well as serial dependence, in fact embody the corresponding structure of the SIM, ie. a, β and e, here representing the "market" as well as the "non – market - induced" returns, respectively.

2.3. Model Testing Results

The tests were performed in two ***Parts*** on monthly basis. Where the model was calibrated on each month within the sample (daily observations) and then tested for prediction capabilities on the following month (daily forecast error was measured)

In ***Part I*** the relationship S&P500/VIX Index was tested in *two separate periods*:

The First Tier - 12 months between 1.1.2005 and 31.12.2005, covering *260 observations* on the daily basis was divided in two parts, ie. January - August, prior to the financial crisis in 2008, and

The Second Tier - 12 months between 1.1.2008 and 31.12.2008 encompassing *262observations* on the daily basis, and divided in two parts ie. January - August, prior to the market meltdown, and September - December for the rest of the year. The model was applied in *two methodological modes*:

▪ ***Mode A***:

the *basic model's version* with the variables, dependent *S&P500 Index,* and coefficients η, representing the constant part of serial dependence, and ρ, the coefficient reflecting the sensitivity of the market index regarding the independent variable *VIX Index*;

▪ ***Mode B***:

the model's basic version was *extended* for a consideration of an additional variable φ representing the serial dependence as the "market memory" and complementing the expected returns' continuity η with the ex post 30 day return determined on the lagged daily basis. Mode B model is model specified in (1), Mode A was added to illustrate the importance of auto regression processes. Using Box-Jenkins residual tests, the significance of only first lag was confirmed, thus the model form used.

The ***First Tier*** of the test (Jan-Dec 2005) indicates for *Mode A,* a range of the *Coefficient of Determination* R^2 2% - 79% with the average value 33%, and the average 68% falling in the range 9% - 90% in the case of *Mode B.* Both peak values of corresponding ranges are attainable at 95% confidence level, assuming the t - distribution. The coefficient η reflecting the constant serial dependence shows the averages 0.95 and 0.22 and the independent variable ρ, as the sensitivity measure of volatility, has results 0.08 and 0.06 in average. The *Standard Error of Estimate* shows a practically identical value for both Modes (0.01). The *D-W statistics* features the average values 2.45 and 0.95, and the short - term forecast expected error *EER* indicates the values 2.45% and 0.95% in average with regard to the corresponding Modes of the model. The summary of average results indicates **Table 1**

Parameter	R^2	η	ρ	AR	D-W	EER %
Jan - Dec Mode A	0.33	0.95	0.08	-_	0.65	2.45
Jan - Dec Mode B	0.68	0.22	0.06	0.69	1.98	0.95

Table 1

Summary of the average results, sample test S&P/VIX Index

First Tier (Jan - Dec 2005)

The ***Second Tier*** of the test on sensitivity S&P500/VIX Index involved the application of both model´s Modes in the period January - December 2008 subsequently divided in two parts, as outlined above.

The results in the whole period tested determined for the basic *Mode A* the coefficient R^2 in the interval 4% - 77% with the average 37%, and for the *Mode B*, the extended version, 9% - 83% with the average 66%. However, here only for the Mode B is the peak value of the quoted range attainable at 95% confidence level of the t - distribution.

The *D-W statistics* varies in the range 0.43 – 1.83, where with the average 0.87 it shows a stronger correlation of residuals ε indicating serial dependence for the basic model, as opposed to the range 1.51 – 2.53, with the average 2.03 for the extended model where the influence of non-volatility-induced factors had been likely taken up by the additional variable φ.

The *EER indicator* concerning the model's predictive accuracy stands more in favour of the extended version with the range 2.1% - 38.3% and average 10.0%, if compared with the range 1.3% - 50.5% and average 13.6% for the basic model, thus offering a stronger explanatory power.

Further, the testing routine encompassed the two ***sub - periods***, i.e. 8 months, January -August 2008, prior to the stock market crash commencing on 15th September, and remaining period of the year, i.e. 4 months, September- December 2008.

The ***first*** sub - period shows for the *Mode A*, the basic model, the *Coefficient of Determination* R^2 smaller in average (46%) than in the case of *Mode B*, the extended model, (71%).

The higher level of correlation concerning the latter can be explained by a complementary effect of the additional variable φ. representing the variable component of the past price series' continuity, measuring this part of the "market memory". The larger coefficient η (0.95 versus 0.85) shows a stronger influence of the constant component of the serial dependence regarding the extended model, but in lieu of a lower level of the S&P500 Index' responsiveness ρ to a change in the VIX Index (0.09 versus 0.12).

The comparison with the ***second*** sub - period highlights a decline in the coefficient ρ for the *Mode A,* indicating a lower level of accuracy concerning the estimates of the expected market index volatility, which can essentially explain a sharp decline in R^2 (from 0.46 to 0.18) in the case of the basic model. This development can be attributed to the unexpected stock market sharp fall.

As far as the *Mode B* is concerned, the fall of R^2 (from 0.71 to 0.55) can be possibly related to a significant decline of the underlying serial dependence factor η, (from 0.95 to 0.80), suggesting a strong incidence of discontinuity of the price series, whilst its variable component φ remained approximately unchanged between both sub - periods. Performance of the *Standard Error of Estimate* appears to be even for both Modes in both sub - periods. The *EER indicator* on the other hand stands distinctly in favour of the Mode B in both sub-periods with substantially lower forecasting percentage errors. The summary of average results then indicates **Table 2**.

Parameter	R^2	η	ρ	AR	SEE	D-W	EER %
Jan-Aug Mode A	0.46	0.85	0.12	_	0.03	0.92	12.35
Sept-Dec Mode A	0.18	0.88	0.02	_	0.06	0.82	16.10
Jan-Aug Mode B	0.71	0.95	0.09	0.64	0.02	2.10	3.42
Sept-Dec Mode B	0.55	0.80	0.09	0.61	0.04	1.88	4.73

Table 2

Summary of average results, sample test S&P500/VIX Index,
Second Tier (Jan - Aug and Sept - Dec 2008)

When comparing results of both model's versions in both time sub- periods it is possible to clearly observe the impact of the additional variable φ. The basic model focuses on the influence of the expected volatility component ρ of the return represented by the VIX Index, with a change in the underlying serial dependence component η being insignificant, whereas the extended model exhibits a strong influence of this component being further reinforced by the variable factor of serial dependence φ, whilst the volatility- determined component ρ shows a decline for the basic model and no change for the extended version.

The general comparison of both models therefore provides for a quantifiable assessment of the two causal elements of the process of generating the returns' expectations, ie. The *volatility* as well as the *serial dependence* concerning the price returns´ performance. It is again possible to observe a substantial impact of the complementary variable φ on the overall significance of the functional relationship. This is clearly indicated by the difference in the coefficient R^2.

The basic version of the model fosters the influence of discontinuity, whereas the extended version emphasizes the impact of the "market memory" with reality, confined to the tested data sample, appearing to lie somewhere in between the two outlined border line results, even though the dispersion of the key results concerning both versions of the model is in some cases rather wide.

However, it appears that the extended model version including the complementary variable φ can offer a more accurate basis for prediction purposes. As a matter of fundamental underlying factor, what appears to make important difference between the model´s performance in the First Tier (2005) and the Second Tier (2008) is a shock pertinent to the market collapse in September 2008 causing a discontinuity in its performance and pointing to the situation of "volatile volatility" based on disjointed investors´ decision-making; since"… *you cannot predict anything with precision. Forecasting volatility is like forecasting the weather. You can measure the intensity and path of a hurricane, and you can calculate the odds of its landing, but,…you cannot predict with confidence exactly it will land and how much damage it will do".* (Mandelbrot and Hudson, 2001, pp. 249).

The ***Part II*** of the VAPM framework testing involved a sample of returns of 10 prime selected companies listed on the NYSE, for their sensitivity concerning the VIX Index as well as the lagged serial dependence represented by the complementary variable φ.

We must note that this test is illustrational in nature because VIX index is not the best measure of volatility for each individual company.

The ***First Tier*** of this part of testing covered on the daily basis one month time period in May 2005. Here the basic model´s *(Mode A)* Coefficient of Determination R^2 ranged from 3.4% (Exxon) to 65.1% (Apple), with the sample average 27.0%. The *EER* forecast error factor reached 3.6% and the *D-W statistics* averaged 0.85. The extended model, *(Mode B)* shows the average R^2 value 55% within the range 80% (Pfizer) and 4.5% (Johnson & Johnson). The *EER* and *D-W* indicators averaged 2.0% and 2.02, respectively.

For the ***Second Tier*** testing the month of September 2008, the coefficient R^2 shows for the *Mode A* the value 26.0% in the range 78.7% (Apple) and 1.0% (IBM), with the *EER* indicator and the *D-W statistics* showing 13.4% and 1.3 values, respectively. The *Mode B* indicates the R^2 average value 40.0% in the range from 2.0% (Hewlett Packard) to 80.0% (Pfizer). The *D-W statistics* showed the average value 1.76. The forecast error *EER* factor indicated the average value 9.9 %.(For summary of results refer to **Table 3**).

Period	R^2	η	ρ	ϕ	EER%	D-W	SE	R^2
5/2005 Model A	0.26	1.13	0.16	-	3.59	0.85	0.03	3% - 65%
5/2005 Model B	0.55	1.57	0.12	0.56	2.4	2.2	0.02	3% - 80%
9/2008 Model A	0.26	0.97	0.09	-	13.42	1.30	0.05	1% - 79%
9/2008 Model B	0.40	0.97	0.08	0.45	9.92	1.76	0.05	4% - 79%

Table 3

Performance of securities' returns/VIX Index

Summary of average results May 2005 & Sept 2008

The model generated a substantial spread of results, nevertheless exhibiting a general viability of the basic relationship consistent with its assumptions. The first period shows a decline in the volatility coefficient ρ, in lieu to a rise of the serial dependence constant η. The comparison of the outlined values confirms again the average significance of the complementary variable φ with its impact on the overall model´s relationship having the constant effect of factor η declined with the ρ influence practically unchanged. The fall in the correlation strength in the second period is also reflected in the increase of forecast error *EER* for both model's versions. This particularly indicates a precision decline concerning the expected volatility assessment due to the unanticipated sharp fall in the market capitalization. This development is also reflected in a substantial fall of the *D-W statistics* indicating an increase in the serial correlation of residuals ε, as a sign of the rise in the effect of the past returns' performance, hence casting doubts on any "random walk" performance.

3. Conclusion

It is possible to say, that only few ideas in the area of modern finance have caused more controversy or held more profound implications than the theory of an efficient market for securities. In an idealized world of "frictionless" markets and costless trading, then prices are supposed to always fully reflect all available information and no extraordinary gains can be obtained. However, this notion has a counter - intuitive and apparently contradictory flavour to it: more random the sequence of price changes generated by a market, hence more "efficient "such a market and therefore the most efficient market of all is the one in which price changes are completely randomly distributed and entirely unpredictable. So ,"*the Random Walk Hypothesis and its close relative, the Efficient Market Hypothesis, have become icons of modern financial economics that continue to fire the imagination of academics and investment professionals alike"* (Lo, MacKinlay,1988, pp. 6), even though a significant body of evidence to the contrary has been mounting over time. Since the onset of development of the Efficient Market Hypothesis, numerous attempts have been launched to capture the gist of the "real" capital market performance. The idea of price returns' distribution normality under conditions of uncertainty underlying the typical models such as the CAPM, Black & Scholes, SIM, MIM, etc., have been gradually overcome by a more realistic view of the capital markets' contemporary behaviour.

Deterministic order in which the future event is uniquely established by the past, in for example the Newtonian sense, is certainly not universal and therefore there is a need to understand also a *disorder.* Chaos, the apparent randomness, as an expression of instability contrasting with regularity is as such found everywhere in the nature and is a part of our own environment, as well as the culture. The revolutionary ideas of for example A.W. Lo, A.C. MacKinlay, S. LeRoy, E. Lucas, and particularly B. Mandelbrot and R. Hudson, have set a new path along which the new, progressive research goes, being reinforced by the markets' turbulent performance observed in the recent years. Mandelbrot for example stated that the dimensions of an object are relative to the view of an observer and may be fractional. He argued that the proportion of information "error" of noise-containing time periods to the error- free periods was constant. An object whose irregularity is constant over different scales as a fractal phenomenon depicted by e.g. the Koch curve and other similar patterns in the model form, can be infinitely extended, whilst embracing a finite space.

The fractal performance of the market environment will have, on the one hand, always remained to some extend unpredictable in terms of its volatility, but, on the other hand, it could be certainly approximated and modelled for example in line with the Mandelbrot's analytical concept underlying the two fundamental factors in forming the market price distribution on the basis of investors' decision making. Broadly speaking, it is the "bearish" fear of loss confining an investor's decision to a reliance on the past performance, ie. the serial dependence on the market "memory", and the "bullish" desire of adventure trying to envisage the future market volatility in order to possibly earn an extraordinary gain at some risk.

On the basis of these outlined ideas, the model VAPM was developed using components of expected volatility and serial dependence to test its relationship with the market index S&P500 as well as selected security returns, in combination with a lagged effect of the underlying autocorrelation of the market returns was developed and applied even under the circumstances of the market implosion in 2008. Results clearly show the more or less inverse but complementing influence of the two enmeshed abovementioned factors basically causing the capital market dynamics. The tests covering various time periods, inspite of certain wider, however broadly explainable result data spreads, generally confirmed the model´s framework viability. Its potential application is envisaged for example in terms of the short term prediction routine, thus perhaps opening an avenue for a further analysis and therefore possibly making a small contribution to the effort of moving research in the area of asset pricing models a step further.

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